Algebra I
University of Helsinki
Department of Mathematics and Statistics
General examination
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1. The group $G = \{a, b, c, d, x, y\}$ has the following mutiplication table:

$\cdot$	a	b	c	d	x	y
a	a	b	c	d	x	y
b	b	c	a	x	y	d
c	c	a	b	y	d	x
d	d	x	y	b	c	a
x	x	y	d	c	a	b
y	y	d	x	a	b	c

   (a) What is the order of b?
   (b) Determine c^{-4}.
   (c) Find a subgroup of G whose order is 4, or show that such subgroup does not exist.

2. The group $G = \{(1), (14), (15), (45), (145), (154)\}$ has subgroups $H = \{(1), (145), (154)\}$ and $K = \{(1), (14)\}$.

   (a) Determine the elements of the coset $(15)H$.
   (b) Is it possible talk about the quotient group G/H? If so, determine the elements and multiplication table of this quotient group.
   (c) Is it possible to talk about the quotient group G/K? If so, determine the elements and multiplication table of this quotient group.

3. Show that the following cancellation property holds in an integral domain D:

   Assume that $a, b, c \in D$ and $a \neq 0$. If $ab = ac$, then $b = c$.

4. Show that the set

$$R = \left\{ \frac{a}{b} : a, b \in \mathbb{Z},\ b \text{ is odd} \right\}$$

   is a subring of $\mathbb{Q}$. What are the units of R?

5. How many homomorphisms there are from the group $\mathbb{Z}_6$ into the group $\mathbb{Z}_4$?