

Osittaisdifferentiaaliyhtälöt
DEMO 9

1. Write down an explicit formula for a solution of

$$\begin{cases} u_t - \Delta u + cu = f & \text{in } \mathbb{R}^n \times (0, \infty); \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}, \end{cases}$$

where $c \in \mathbb{R}$.

2. Let $\phi : \mathbb{R} \rightarrow \mathbb{R}$ be smooth and convex. Assume that u solves the heat equation (that is, $u_t - \Delta u = 0$) and $v = \phi(u)$. Prove that v is a subsolution of the heat equation, i.e. $v_t - \Delta v \leq 0$.

3. Prove that $v = |Du|^2 + u_t^2$ is a subsolution of the heat equation, where u solves the heat equation.