Osittaisdifferentiaaliyhtälöt
DEMO 6

1. Let $B^+ = \{x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n : |x| < 1, x_n > 0\}$ be the open upper half ball. Assume that $u \in C^2(B^+) \cap C(\overline{B^+})$ is harmonic in B^+, with $u = 0$ on $\partial B^+ \cap \{x_n = 0\}$. Set

$$v(x) = \begin{cases} u(x) & \text{if } x_n \geq 0; \\ -u(x_1, \ldots, x_{n-1}, -x_n) & \text{if } x_n < 0. \end{cases}$$

for $x = (x_1, x_2, ..., x_n) \in B(0,1)$. Prove that v is harmonic in $B(0,1)$.

2. Let $g \in C(\mathbb{R}^{n-1})$ be bounded. Show that

$$u(x) = \frac{2x_n}{n\alpha_n} \int_{\mathbb{R}^{n-1}} \frac{g(y)}{[(x_1 - y_1)^2 + ... + (x_{n-1} - y_{n-1})^2 + x_n^2]^{n/2}} dy$$

is the unique bounded, continuous solution of

$$\begin{cases} \Delta u = 0 & \text{in } \mathbb{R}^n_+; \\ u = g & \text{on } \partial\mathbb{R}^n_+. \end{cases}$$