Osittaisdifferentiaaliyhtälöt
DEMO 5

1. Prove the following comparison principle: suppose that $u, v \in C^2(\Omega) \cap C(\bar{\Omega})$ satisfy $\Delta u \leq \Delta v$ in Ω. If $v \leq u$ on $\partial\Omega$, then $v \leq u$ in Ω.

2. Suppose that $u : \mathbb{R}^n \to \mathbb{R}$ is harmonic and bounded from above (there is $M < \infty$ such that $u(x) \leq M$ for all $x \in \mathbb{R}^n$). Show that u is a constant

3. Let u be a non-negative harmonic function in $\Omega = B(0,1) \setminus \{0\} \subset \mathbb{R}^n$. Show that there is a constant c, depending only on n, such that

$$\max_{\partial B(0,r)} u \leq c \min_{\partial B(0,r)} u,$$

for all $0 < r \leq 1/2$.

4. Show that Green's function is non-negative.