**Osittaisdifferentiaaliyhtälöt**
**DEMO 3**

1. Write down an explicit formula for a function $u$ solving

$$\begin{cases} u_t + b \cdot Du + cu = 0 & \text{in } \mathbb{R}^n \times (0,\infty); \\ u = g & \text{on } \mathbb{R}^n \times \{t = 0\}. \end{cases}$$

Here $c \in \mathbb{R}$ and $b \in \mathbb{R}^n$ are constants.

2. Let $u \in C^1(\mathbb{R}^n)$ and

$$\varphi(r) = ⨍_{\partial B(x,r)} u(y)\, dS(y).$$

Prove that

$$\varphi'(r) = ⨍_{\partial B(0,1)} Du(x+ry) \cdot y\, dS(y).$$

3. Find a solution to the following boundary value problem

$$\begin{cases} \Delta u = u^3 & \text{in } B(0,1); \\ u = 0 & \text{on } \partial B(0,1). \end{cases}$$

4. Find a solution to the following boundary value problem

$$\begin{cases} \Delta u = 0 & \text{in } B(0,1); \\ u = 1 & \text{on } \partial B(0,1). \end{cases}$$

5. Find a solution to the following boundary value problem

$$\begin{cases} \Delta u = 1 & \text{in } \Omega = \{x \in \mathbb{R}^3 : a < |x| < b\}; \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $0 < a < b < \infty$.

6. Prove that Laplace's equation

$$\Delta u = 0$$

is rotation invariant; that is, if $O$ is an orthogonal $n \times n$ matrix and we define

$$v(x) = u(Ox),$$

then

$$\Delta v = 0.$$