Osittaisdifferentiaaliyhtälöt
DEMO 2

In problems 1–5, solve: $u = u(x, y)$.

1.
$$\begin{cases} 2u_x + 3u_y = 0; \\ u(0, y) = \sin y. \end{cases}$$

2.
$$\begin{cases} 2u_x + 3u_y = x; \\ u(0, y) = y. \end{cases}$$

3.
$$\begin{cases} 2u_x + 3u_y = xu; \\ u(0, y) = y. \end{cases}$$

4.
$$\begin{cases} 2u_x + 3u_y = xu + x; \\ u(0, y) = y. \end{cases}$$

5.
$$u_x + xu_y = 0.$$

6. Let f and g be two given continuous functions and let c be a constant. Solve the following initial value problem:

$$\begin{cases} f(y)u_x + u_y = cu; \\ u(x, 0) = g(x). \end{cases}$$

[Hint: the solution should be of the form $u(x, y) = e^{cy} g\big(x - \int_0^y f(z)\, dz\big)$.]

7. Suppose that $u \in C^1(\mathbb{R}^2)$ is a solution of

$$a(x, y)u_x + b(x, y)u_y = 0.$$

Show that for arbitrary $H \in C^1(\mathbb{R})$ also $H(u)$ is a solution.