Osittaisdifferentiaaliyhtälöt
DEMO 1

1. Classify each of the PDEs given as examples in the lecture as follows: Is the PDE linear, semilinear, quasilinear or fully nonlinear? What is the order of the PDE?

2. Suppose that $u \in C^1(\mathbb{R}^n)$ and $\varphi \in C_0^1(\mathbb{R}^n)$. Prove that

$$\int_{\mathbb{R}^n} u \partial_k \varphi \, dx = -\int_{\mathbb{R}^n} \varphi \partial_k u \, dx, \quad k = 1, 2, \ldots, n.$$

3. Let $u(x) = |x|^\alpha, x \in \mathbb{R}^n, x \neq 0$, where $\alpha \in \mathbb{R}$. Calculate $\Delta u(x)$.

4. Let u be a continuous function in $\Omega \subset \mathbb{R}^n$. Suppose that

$$\int_\Omega u\varphi \, dx = 0$$

for all $\varphi \in C_0^\infty(\Omega)$. Show that $u(x) = 0$ for all $x \in \Omega$.

5. Prove that if f is continuous, then

$$f(x) = \lim_{r \to 0} ⨍_{B(x,r)} f \, dy = \lim_{r \to 0} ⨍_{\partial B(x,r)} f \, dS.$$

6. Consider the Neumann problem:

$$\begin{cases} \Delta u(x) = f(x), & x \in \Omega; \\ \frac{\partial u}{\partial \nu}(x) = 0, & x \in \partial\Omega. \end{cases} \tag{1}$$

Show that either equation (1) does not have a solution or it has many solutions.

7. Prove that if equation (1) has a solution $u \in C^2(\Omega) \cap C^1(\bar{\Omega})$, then

$$\int_\Omega f \, dx = 0.$$