HARMONIC ANALYSIS
2016

9. Homework sheet
24.11.2016

9.1. **Homework.** Prove Theorem 8.10 paying special attention to the constant c_p. Find an estimate for c_p.

9.2. **Homework.** Suppose that $f \in L^1_{\text{loc}}(\mathbb{R}^n)$ and $1 < p < \infty$. Find a value for the constant $A_{n,p}$ in the Fefferman-Stein inequality

$$\big|\big|M_\Delta f\big|\big|_{L^p(\mathbb{R}^n)} \leq A_{p,n}\big|\big|M^{\#}_\Delta f\big|\big|_{L^p(\mathbb{R}^n)}$$

when in the proof for the relative distributional inequality we choose $b = 1/2$ and $c = 2^{-n-2-p}$.

9.3. **Homework.** Let $f \in L^p(\mathbb{R}^n)$, $1 < p < \infty$. Give an example which shows that the result

$$\big|\big|M_\Delta f\big|\big|_{L^p(\mathbb{R}^n)} \leq A_{p,n}\big|\big|M^{\#}_\Delta f\big|\big|_{L^p(\mathbb{R}^n)}$$

is stronger than the inequality

$$\big|\big|f\big|\big|_{L^p(\mathbb{R}^n)} \leq C_{p,n}\big|\big|M^{\#} f\big|\big|_{L^p(\mathbb{R}^n)} .$$

9.4. **Homework.** A localized version of the John-Nirenberg inequality which is restricted to dyadic cubes:
Suppose that f is an integrable function on a dyadic cube Q_0. Suppose that

$$\frac{1}{|Q|}\int_Q |f(x) - f_Q|\, dx \leq 1$$

holds for all dyadic subcubes $Q \subset Q_0$. Show that there exist positive constant c_1 and c_2 so that for all $\lambda > 0$

$$\big|\big\{x \in Q_0 : |f(x) - f_{Q_0}| > \lambda\big\}\big| \leq c_1 \exp(-c_2\lambda)|Q_0| .$$

Hint: Apply the relative distributional inequality with $c = 1/\lambda$ for $h(x) = (f - f_{Q_0})\chi_{Q_0}$, noting that $M^{\#}_\Delta h(x) \leq 1$. Study the cases $\lambda \geq 2^{n+1}$ and $\lambda \leq 2^{n+1}$ separately.

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