

REAL-VARIABLE HARMONIC ANALYSIS I
2016

6. HOMEWORK SHEET
20.10.2016

6.1. **Homework.** If $f \in L \log L(\mathbb{R}^n)$, show that $Mf \in L^1_{\text{loc}}(\mathbb{R}^n)$.

6.2. **Homework.** We proved the C. Fefferman-E. M. Stein inequality for functions $f \in L^1(\mathbb{R}^n)$. If $f \in L^1_{\text{loc}}(\mathbb{R}^n)$, show that the C. Fefferman-E. M. Stein inequality is valid then also.

6.3. **Homework.** If $f : \mathbb{R}^n \rightarrow \mathbb{R}$, $f(x) = \chi_{B_R(0)}(x)$, show that

$$c_1(n) \frac{R^n}{(|x| + R)^n} \leq Mf(x) \leq c_2(n) \frac{R^n}{(|x| + R)^n}$$

for some $c_1(n) > 0$ and $c_2(n) > 0$.

6.4. **Homework.** Let f be the characteristic function of a ball $B(0, R)$ from Homework 6.3. If $g(x) = Mf(x)$, find an upper estimate for $Mg(x)$.