

**REAL-VARIABLE HARMONIC ANALYSIS I**  
**2016**

3. HOMEWORK SHEET

29.9.2016

3.1. **Homework.** Let  $1 \leq p_1 < p < p_2 < \infty$ . Let  $f \in L^p$ . Show that the functions

$$f_1 = f \mathbf{1}_{\{|f|>\alpha\}} \in L^{p_1} \text{ and } f_2 = f \mathbf{1}_{\{|f|\leq\alpha\}} \in L^{p_2}.$$

3.2. **Homework.** Prove the Dyadic Vitali-type Covering Lemma.

3.3. **Homework.** Show the pointwise inequality

$$M_\Delta f(x) \leq c(n)Mf(x) \text{ for all } x \in \mathbb{R}^n.$$

3.4. **Homework.** Give an example of a function  $f$  which shows that there is no constant  $c \in (0, \infty)$  such that

$$Mf(x) \leq cM_\Delta f(x) \text{ for all } x \in \mathbb{R}.$$

3.5. **Homework.** If  $0 < \alpha < n$  and  $\delta > 0$ , show that there is a constant  $c(n, \alpha)$  so that

$$\int_{B_\delta(x)} \frac{|f(y)|}{|x-y|^{n-\alpha}} dy \leq c(n, \alpha)\delta^\alpha Mf(x) \text{ for all } x \in \mathbb{R}^n.$$