REAL-VARIABLE HARMONIC ANALYSIS I
2016

2. Homework sheet
22.9.2016

2.1. **Homework.** Let $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \chi_{(-1,1)}(x)$. Find $Mf(x)$.

2.2. **Homework.** Let $f : \mathbb{R}^n \to \mathbb{R}$, $f(x) = \chi_{B_1(0)}(x)$. Find $Mf(x)$.

2.3. **Homework.** Show that the Hardy-Littlewood maximal operator commutes with translations and dilations.

2.4. **Homework.** Let $\mathcal{F}$ be a family of balls in $\mathbb{R}^n$ of bounded diameter. Show that for every $\epsilon > 0$ there exits a countable subcollection $\mathcal{G}_\epsilon \subset \mathcal{F}$ of pairwise disjoint balls such that

$$\bigcup_{B \in \mathcal{F}} B \subset \bigcup_{B \in \mathcal{G}_\epsilon} (3+\epsilon)B\,.$$

2.5. **Homework.** Let $1 \leq p_1 < p_2 < \infty$. If

$$f \in L^{p_1,\infty}(\mathbb{R}^n) \cap L^{p_2,\infty}(\mathbb{R}^n)\,,$$

show that $f \in L^p(\mathbb{R}^n)$ for every $p \in (p_1, p_2)$.

Date: 10092016/RHS.