# HARMONIC ANALYSIS 2016

## 10. Homework sheet 8.12.2016

10.1. **Homework.** Homework 9.4.

10.2. **Homework.** Suppose that $1 < p < \infty$. Give an example of a function $f \in L^{p,\infty}(Q_0)$, but $K_p^p(f, Q_0) = \infty$.

10.3. **Homework.** Suppose that $1 < p < \infty$ is given, $Q_0$ is a finite cube in $\mathbb{R}^n$, and $b \in (0, 2^{-n})$. If $f \in L^1(Q_0)$ is given such that $K_p^p(f, Q_0) < \infty$, show that the inequality

$$\begin{aligned} &\big|\{x \in Q_0 : M_\Delta(f - f_{Q_0})(x) > \lambda\}\big| \\ &\le \frac{aK_p(f, Q_0)}{\lambda}\big|\{x \in Q_0 : M_\Delta(f - f_{Q_0})(x) > b\lambda\}\big|^{(p-1)/p} \end{aligned}$$

holds for $a = (1 - 2^n b)^{-1}$ and for all $\lambda$ whenever $b\lambda \ge |Q_0|^{-1} \int_{Q_0} |f(x) - f_{Q_0}| \, dx$.

10.4. **Homework.** When $n$ and $p$ are given, estimate the constant $c(n, p)$ in the proof for the John-Nirenberg $2^{\text{nd}}$ theorem.

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