

REAL-VARIABLE HARMONIC ANALYSIS I
2016

1. HOMEWORK SHEET

15.9.2016

Remark The functions $Mf(x)$ and $M_0f(x)$ are defined as in lectures.

1.1. **Homework.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2$. Find $Mf(x)$.

1.2. **Homework.** Let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = \chi_{(0,1)}(x)$. Find $Mf(x)$.

1.3. **Homework.** Show that

$$2^{-n}M_0f(x) \leq Mf(x) \leq M_0f(x).$$

1.4. **Homework.** Prove that the centered Hardy-Littlewood maximal function $Mf(x)$ is lower semicontinuous.

1.5. **Homework.** Let (X, μ) be a measure space. Let $1 \leq p < \infty$. Show that

$$\|f\|_{L^p} = \left(\int_0^\infty p\alpha^{p-1} \lambda_f(\alpha) d\alpha \right)^{1/p}.$$