

**Eulerin  $\Gamma$ - ja  $B$ -funktiot, Riemannin  $\zeta$ -funktio, ja niiden ominaisuuksia:**

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt \quad \Gamma(n+1) = n! \quad \Gamma(z+1) = z\Gamma(z) \quad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$$

$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z} \quad B(p, q) = \int_0^1 t^{p-1}(1-t)^{q-1} dt \quad B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

**Fourier'n sarjat:**

Kun  $f$  on  $L$ -periodinen:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{i2\pi kx/L} \quad c_k = \frac{1}{L} \int_0^L f(x) e^{-i2\pi kx/L} dx$$

Kun  $f$  on  $2\pi$ -periodinen:

$$f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx} \quad c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx$$

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(kx) + b_k \sin(kx)) \quad a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx) dx \quad b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(kx) dx$$

**Fourier'n muunnos ja käänteismuunnos, sekä ominaisuuksia:**

$$\mathcal{F}[f] = \int_{-\infty}^{\infty} f(x) e^{-ipx} dx \quad f(x) = \int_{-\infty}^{\infty} \mathcal{F}[f] e^{ipx} \frac{dp}{2\pi} \quad f * g = \int_{-\infty}^{\infty} f(x-y)g(y) dy$$

$$\mathcal{F}_s[f] = \int_0^{\infty} f(x) \sin(px) dx \quad f(x) = \frac{2}{\pi} \int_0^{\infty} \mathcal{F}_s[f] \sin(px) dp \quad \mathcal{F}[f * g] = \mathcal{F}[f]\mathcal{F}[g]$$

$$\mathcal{F}_c[f] = \int_0^{\infty} f(x) \cos(px) dx \quad f(x) = \frac{2}{\pi} \int_0^{\infty} \mathcal{F}_c[f] \cos(px) dp \quad \mathcal{F}[f'] = ip\mathcal{F}[f]$$

$$\mathcal{F}[f](\mathbf{p}) = \int_{\mathbb{R}^d} d^d \mathbf{x} f(\mathbf{x}) e^{-i\mathbf{p} \cdot \mathbf{x}} \quad f(\mathbf{x}) = \int_{\mathbb{R}^d} \frac{d^d \mathbf{p}}{(2\pi)^d} \mathcal{F}[f] e^{i\mathbf{p} \cdot \mathbf{x}}$$

**Parsevalin kaava Fourier'n sarjoille ja muunnokselle:**

$$\int_{-\pi}^{\pi} |f(x)|^2 dx = 2\pi \sum_{k=-\infty}^{\infty} |c_k|^2 = \pi \left[ \frac{|a_0|^2}{2} + \sum_{k=1}^{\infty} (|a_k|^2 + |b_k|^2) \right] \quad \int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\mathcal{F}[f]|^2 \frac{dp}{2\pi}$$

**Laplacen muunnos ja käänteismuunnos, sekä ominaisuuksia:**

$$\mathcal{L}[f] = \int_0^{\infty} e^{-st} f(t) dt \quad f = \int_{\sigma-i\infty}^{\sigma+i\infty} \mathcal{L}[f] e^{st} \frac{ds}{2\pi i} \quad \mathcal{L}[af + bg] = a\mathcal{L}[f] + b\mathcal{L}[g]$$

$$\mathcal{L}[tf] = -\frac{d\mathcal{L}[f]}{ds} \quad \mathcal{L}[f'] = s\mathcal{L}[f] - f(0^+) \quad \mathcal{L} \left[ \int_0^t f(\tau)g(t-\tau) d\tau \right] = \mathcal{L}[f]\mathcal{L}[g]$$