Finite model theory
Problems 9
Tuesday 15.11.2016

1. Construct a sentence φ of IFP such that for all finite *ordered* structures $\mathfrak{A}$:
$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is even.}$$

2. Let τ be relational and finite. Show that for any $\varphi \in \mathrm{PFP}[\tau]$ there is an *equivalent* sentence $\varphi^* \in \mathcal{L}^{\omega}_{\infty,\omega}[\tau]$, that is, for all finite τ-models $\mathfrak{A}$:
$$\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models \varphi^*.$$

3. Let $\tau = \emptyset$. Show that there is no $\varphi \in \mathrm{PFP}[\tau]$ such that for all finite $\mathfrak{A}$:
$$\mathfrak{A} \models \varphi \Leftrightarrow |\mathrm{Dom}(\mathfrak{A})| \text{ is even.}$$

4. Let τ be a finite unary vocabulary. Show that for every $\varphi \in \mathrm{PFP}[\tau]$ there is $\varphi^* \in \mathrm{FO}[\tau]$ such that for all finite τ-models $\mathfrak{A}$:
$$\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models \varphi^*.$$

5. Construct a sentence $\varphi \in \mathcal{L}^{\omega}_{\infty,\omega}[\tau]$, where τ finite, that is not equivalent to any $\psi \in \mathrm{PFP}[\tau]$.

6. Show that on *ordered* finite structures, SO $\leq$ PFP.