Finite model theory

Problems 8

Tuesday 8.11.2016

1. A bipartite graph $\mathbb{G}$ is balanced if there is $U \subseteq \text{Dom}(\mathbb{G})$ such that $|U| = |\text{Dom}(\mathbb{G}) \setminus U|$ and all edges of $\mathbb{G}$ are between elements of U and $\text{Dom}(\mathbb{G}) \setminus U$. Show that there is no sentence $\varphi \in \text{MSO}$ such that for all finite graphs $\mathbb{G}$:

$$\mathbb{G} \models \varphi \Leftrightarrow \mathbb{G} \text{ is balanced.}$$

2. Construct a sentence $\varphi \in \text{MSO}$ such that for all finite graphs $\mathbb{G}$:

$$\mathbb{G} \models \varphi \Leftrightarrow \mathbb{G} \text{ is 3-colorable.}$$

(The class of 3-colorable graphs in not known to in PTIME.)

3. Let $L \subseteq \Sigma^+$ be definable in MSO. Show that L is in PTIME.

4. Formulate (informally) an EF-game for MSO. Let $\tau = \{P_1, ..., P_l\}$, be a unary vocabulary, and let $\mathfrak{A}$ and $\mathfrak{B}$ be τ-models such that $\mathfrak{A} \equiv_{2^k} \mathfrak{B}$ (FO). Show using the EF-game that $\mathfrak{A} \equiv_k \mathfrak{B}$ (MSO).

5. Let τ be as in the previous exercise. Show that for every $\varphi \in \text{MSO}[\tau]$ there is $\varphi^* \in \text{FO}[\tau]$ such that for all finite τ-models $\mathfrak{A}$:

$$\mathfrak{A} \models \varphi \Leftrightarrow \mathfrak{A} \models \varphi^*.$$