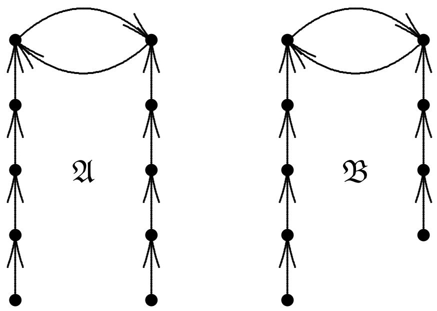
Finite model theory

Problems 5

Tuesday 11.10.2016

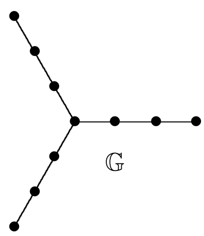
1. Let R be a binary relation and let $\mathfrak{A}$ and $\mathfrak{B}$ be the following $\{R\}$-models. Determine the smallest k such that there exists a formula φ of $\mathcal{L}^k_{\infty,\omega}$ that separates $\mathfrak{A}$ and $\mathfrak{B}$ (i.e., φ holds in exactly one of the models).



2. Let $\mathfrak{A}$ be a τ model. A sentence $\varphi \in \mathrm{FO}[\tau]$ is called a *Scott sentence* of $\mathfrak{A}$, if $\mathfrak{A} \models \varphi$ and for all τ models $\mathfrak{B}$, if $\mathfrak{B} \models \varphi$ then $\mathfrak{A} \cong \mathfrak{B}$. Furthermore, the *Scott Height* $\mathrm{SH}(\mathfrak{A})$ of $\mathfrak{A}$ is defined by

$$\mathrm{SH}(\mathfrak{A}) = min(\{qr(\varphi) \mid \varphi \text{ is a Scott sentence of } \mathfrak{A}\} \cup \{\infty\})$$

Determine the Scott Height of the following graph $\mathbb{G}$



3. Show that for every $r \in \mathbb{N}$ there is $n \in \mathbb{N}$ such that $\mathbb{G}_n \cong_r \mathbb{G}'_n$, where

- $Dom(\mathbb{G}_n) = \{0, ..., n-1\}$ and $Dom(\mathbb{G}'_n) = \{0, ..., 2n-1\}$
- $E^{\mathbb{G}_n} = \{(k, k+1) \mid k \in \{0, ..., n-2\}\} \cup \{(n-1, 0)\}$
- $E^{\mathbb{G}'_n} = E^{\mathbb{G}_n} \cup \{(k, k+1) \mid k \in \{n, ..., 2n-2\}\} \cup \{(2n-1, n)\}$

4. Show that connectivity of finite graphs (that is, every pair of vertices is connected by a path) cannot be defined in first-order logic.

5. Show that connectivity of finite graphs cannot be defined $\mathcal{L}^2_{\infty,\omega}$.

6. Construct a sentence of $\mathcal{L}^3_{\infty,\omega}$ that expresses connectivity of finite graphs.