Finite model theory

Problems 2

Tuesday 20.9.2016

1. Let τ be a finite vocabulary without function symbols. Determine the number of atomic τ-sentences.

2. Let $\mathfrak{A}$ and $\mathfrak{B}$ be τ-models, p a partial isomorphism from $\mathfrak{A}$ to $\mathfrak{B}$, and $a_1, ..., a_k \in \mathrm{dom}(p)$. Show that

$$\mathfrak{A} \models \varphi[a_1/x_1, ..., a_k/x_k] \Leftrightarrow \mathfrak{B} \models \varphi[p(a_1)/x_1, ..., p(a_k)/x_k],$$

for all atomic $\tau \cup \{x_1, ..., x_k\}$-sentences φ.

3. Let $\{E\}$ be a binary relation symbol. Construct a first-order sentence φ such that

$$\mathfrak{A} \models \varphi \Leftrightarrow E^{\mathfrak{A}} \text{ is an equivalence relation.}$$

4. Let τ be a finite vocabulary without function symbols, and $\mathfrak{A}$ a finite τ-model. Show that there exists a first-order sentence $\varphi_{\mathfrak{A}}$ such that for all $\mathfrak{B}$ the following holds:

$$\mathfrak{B} \models \varphi_{\mathfrak{A}} \Leftrightarrow \mathfrak{B} \cong \mathfrak{A}.$$

5. Let f be a unary function symbol. Construct a $\{f\}$-sentence φ such that the following holds for all $\mathfrak{A}$ of vocabulary $\{f\}$:

$$\mathfrak{A} \models \varphi \Rightarrow \mathrm{Dom}(\mathfrak{A}) \text{ is infinite.}$$

6. Let $\mathfrak{A}_n$ be a structure of vocabulary $\{+\}$ (binary function symbol) such that $\mathrm{Dom}(\mathfrak{A}_n) = \{0, ..., n-1\}$ and $+$ is interpreted as addition modulo n. Give an example of a sentence φ such that $\mathfrak{A}_{13} \models \varphi$ but $\mathfrak{A}_{17} \not\models \varphi$.