EVOLUTION AND THE THEORY OF GAMES

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Exercises 05-10-2016

11. The *Size Game* is related to the *War of Attrition*, but there is an important difference. In the War of Attrition, the actual cost to both players is determined by the looser. As a consequence, the winner always pays less than he was prepared to pay. In the Size Game, however, the investment is made in advance of the actual contest, and so the cost is paid in advance, and there is no refund to the winner.

(a) Give an expression for $E(c', c)$, i.e., the payoff to c' against c, where c' and c are the advance payments.
(b) Is there an pure ESS? Does there exist a mixed ESS with a probability density?
(c) How, do you guess, will the pre-game investment evolve in the long run, i.e., as a consequence of many successive mutation-invasion events?
(Hint: in the (c', c)-plane, indicate which c' can invade which c.)

12. Consider the asymmetric *Battle of the Sexes* or *Who Takes Care of the Kids?* with the modified payoff matrix

	Stay	Run
Stay	$V-C$, $V-C-\gamma$	$V-2C$, $V-\gamma$
Run	V , $V-2C-\gamma$	0 , $-\gamma$

(c.f. lecture notes 10-11-2011, section 18)

(a) Interpret the V, C and γ in biological terms.
(b) Give all ESSs of the game.

13. Consider the asymmetric *Hawk-Dove* game with payoff matrix

	H	D
H	$\theta R-(1-\theta)C$, $(1-\theta)R-\theta C$	R , 0
D	0 , R	$\frac{1}{2}R$, $\frac{1}{2}R$

where $\frac{1}{2} \leq \theta \leq 1$ is the probability that the row-player wins a H×H contest. You know that this game has to be solved using conditional strategies. You also know that this presumes that both players know in which role the are: row-player

("strong") or column-player ("weak"). To know one's role, one has to do some sort of assessment. Suppose this assessment has a cost. Will it then still be worthwhile to bother? Maybe playing unconditional Hawk or unconditional Dove in order to avoid the cost of assessment will beat the assessor strategy.

To find that out, we play the *Hawk-Dove-Assessor* game. This is a symmetric game with strategies *always Hawk* (aH), *always Dove* (aD) – both these strategies ignore roles – and the *Assessor* (A), who plays Hawk as row-player but Dove as column-player, but who always pays an extra cost γ for doing the assessment.

(a) Complete the following payoff matrix for the symmetric *Hawk-Dove-Assessor* game:

	aH	aD	A
aH			
aD			
A			

(payoffs to the row-player)

(b) Indicate in the (γ, θ)-plane where which pure strategy (aH, aD or A) is an ESS. In this exercise we do not consider mixed strategies.