Mathematics, photography, and harmonic image inpainting

Samuli Siltanen

Applications of matrix computations
9.9.2016 University of Helsinki

Outline


PRISMA


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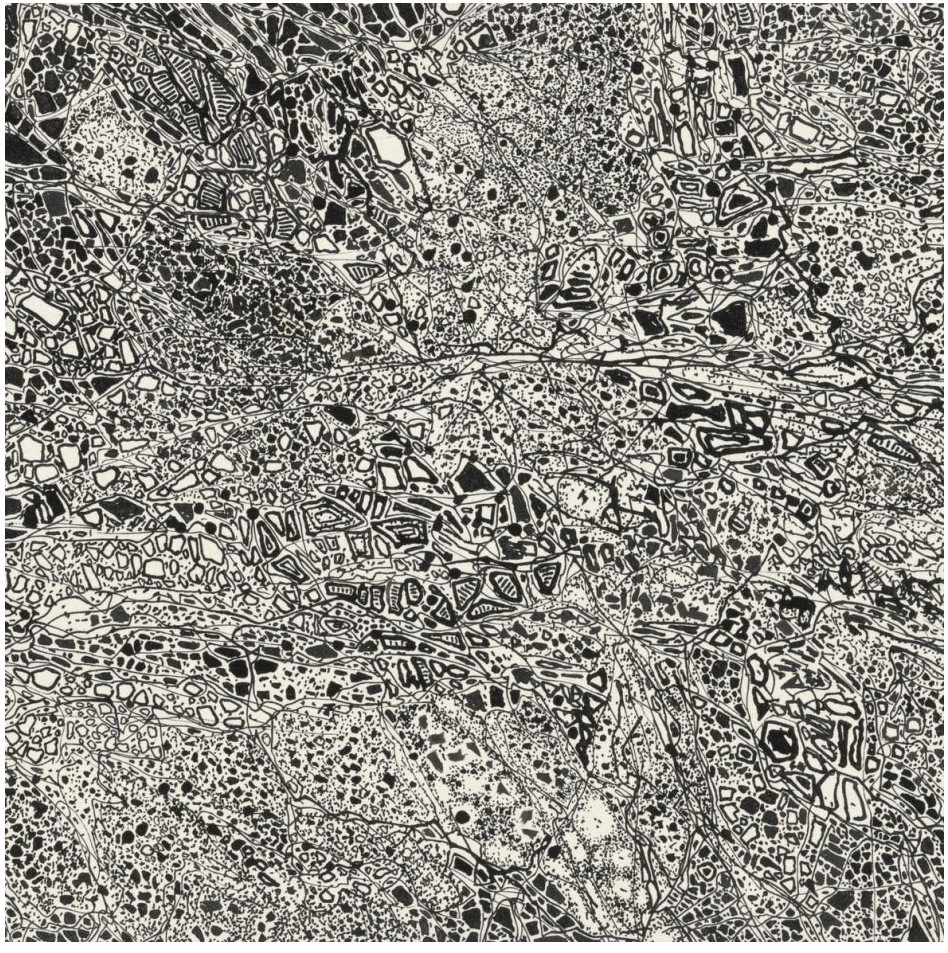
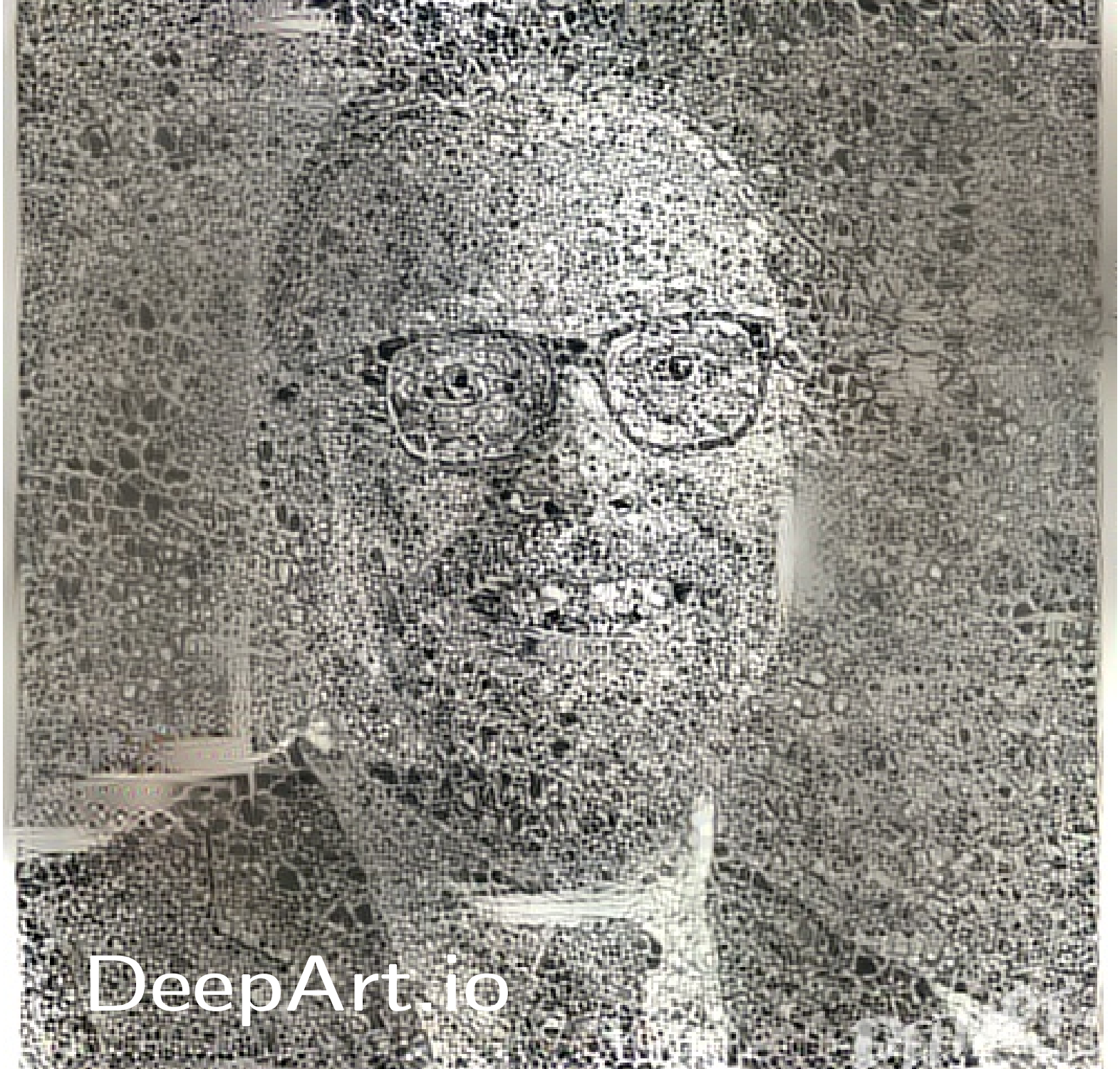


Jean Dubuffet


DeepArt.io

Katsura Funakoshi


DeepArt.io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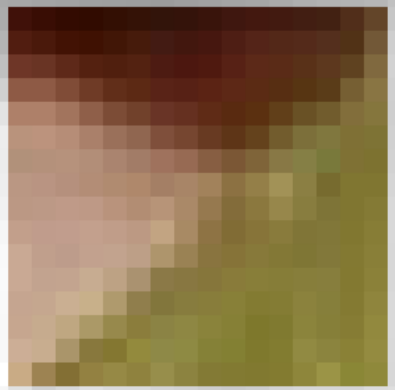
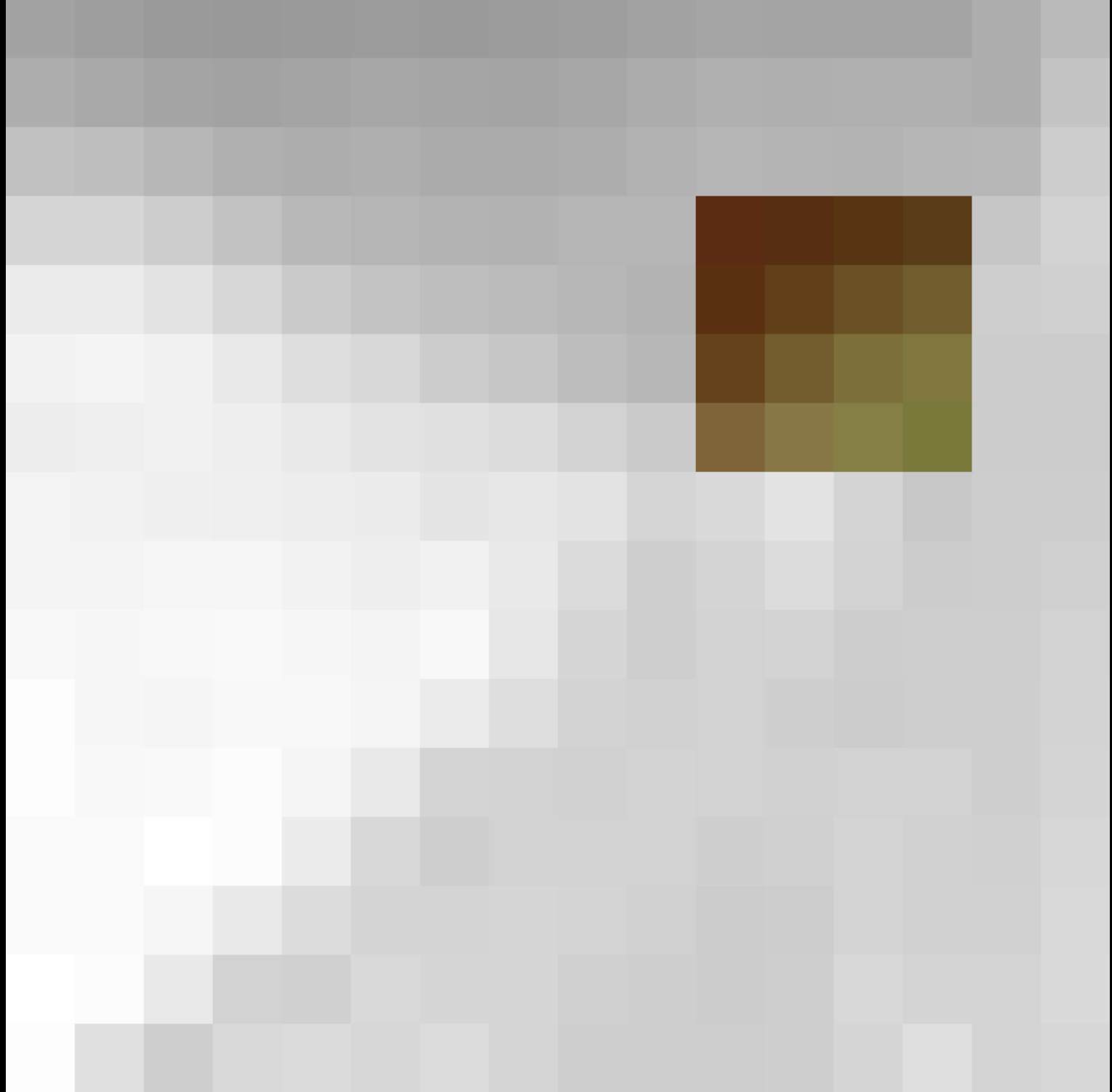
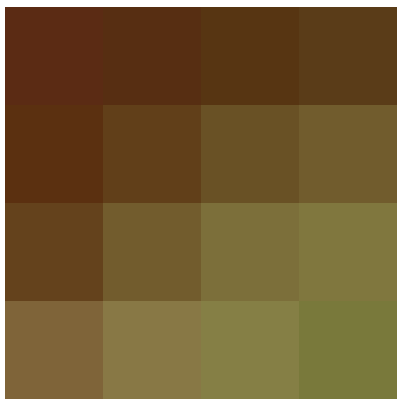
PRISMA


DeepArt.io

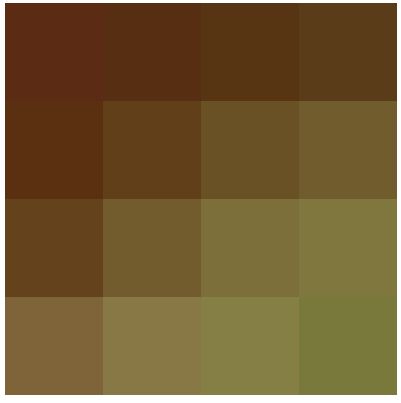
Outline







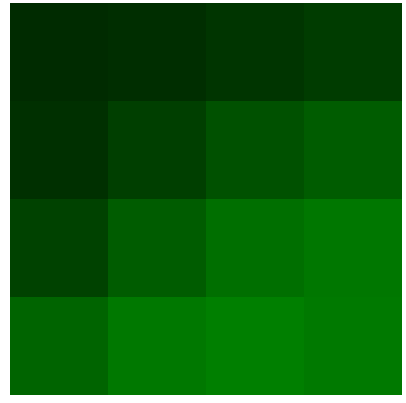
Color channels (värikanavat)



R

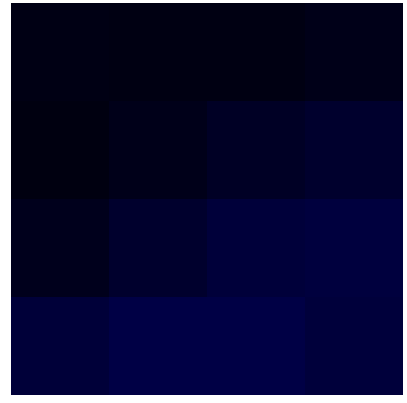
91	87	87	90
91	97	105	113
100	114	124	128
127	136	133	121

G



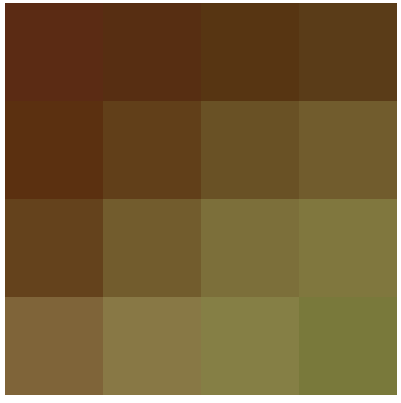
43	46	53	60
48	63	81	92
66	92	111	119
100	120	127	121

B

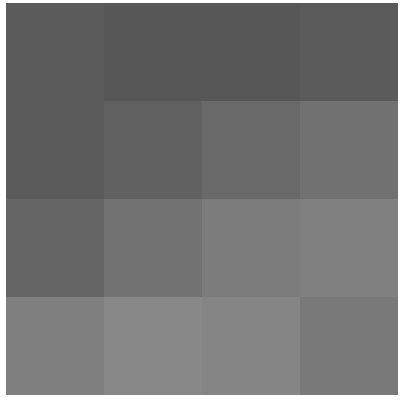


20	18	18	24
16	25	37	45
28	45	58	62
57	69	69	59

Color channels (värikanavat)



R



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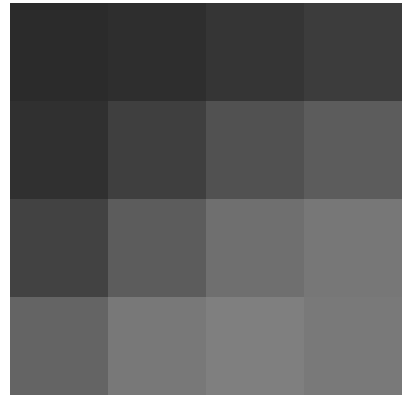

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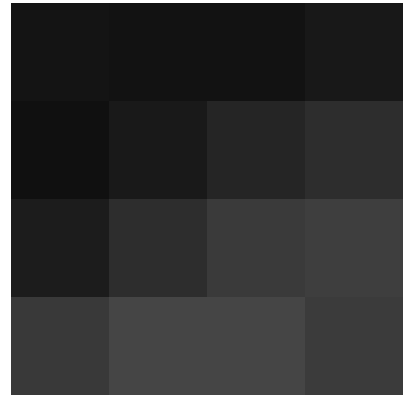

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Outline

Let us study a few very basic mathematical operations related to enhancement of photos

Gamma correction is a simple yet powerful technique for brightening or darkening photos. Almost every conference talk I see would benefit from gamma correcting some of the images in the slides.

To apply gamma correction, it is a good idea to scale the image so that the pixel values are floating point numbers between zero and one. Then each pixel value is raised to power γ.

If $0 < \gamma < 1$ then the image becomes brighter.
If $\gamma > 1$, the image gets darker.

Outline

Elias Garcia Martinez: *Ecce Homo*

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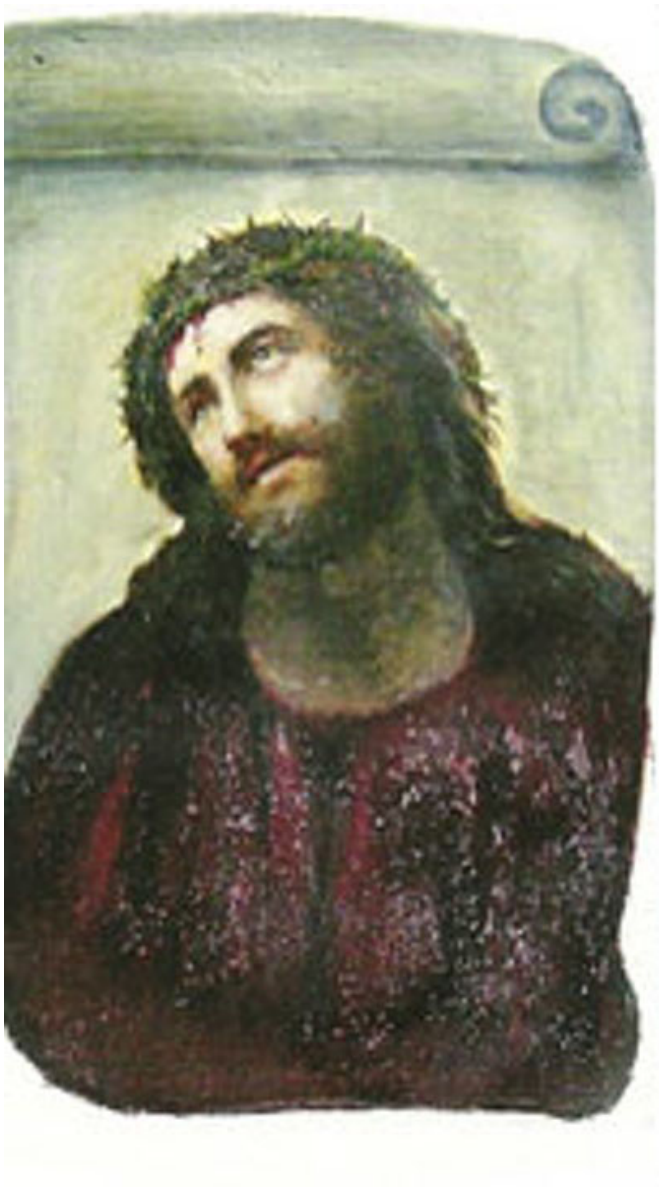

Elias Garcia Martinez: *Ecce Homo* manual inpainting by Cecilia Giménez


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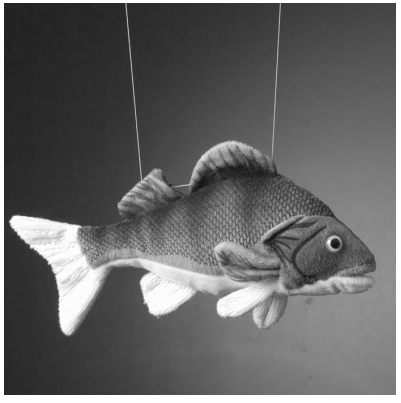
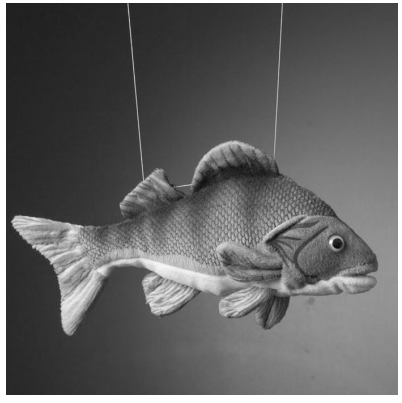
Color channels (värikanavat)

R G B

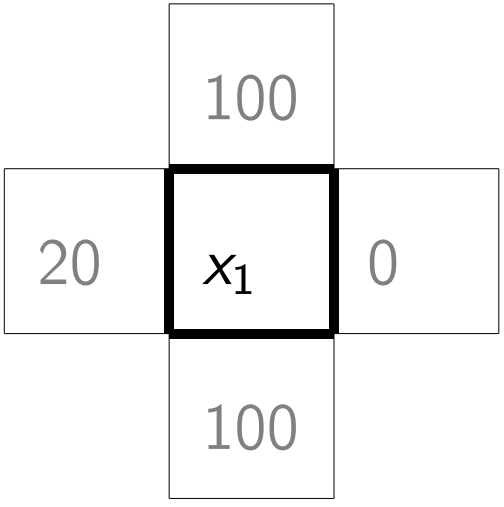





Inpainting: fill in numbers in the middle when the numbers at the boundary are known

Mathematical view of the inpainting problem

	100	100	100	100	100	100	100	100	
20									0
20									0
20									0
20									0
20									0
20									0
	100	100	100	100	100	100	100	100	

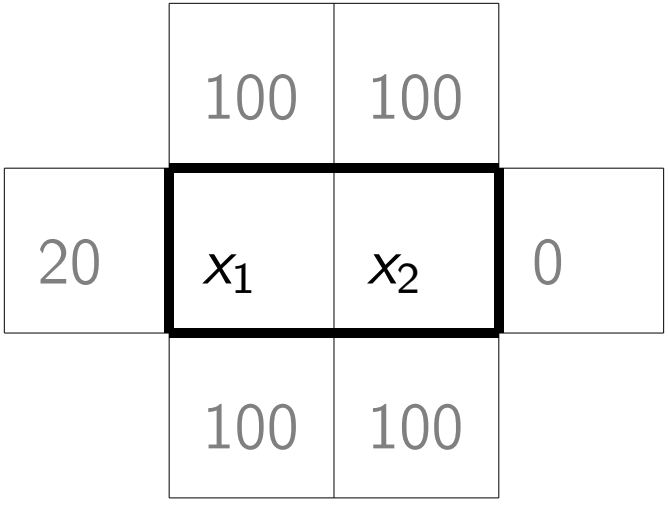
The simplest case.
Yksinkertaisin tilanne.



Solution: take the average (otetaan keskiarvo):

$x_1 = (100 + 0 + 100 + 20)/4 = 220/4 = 55.$

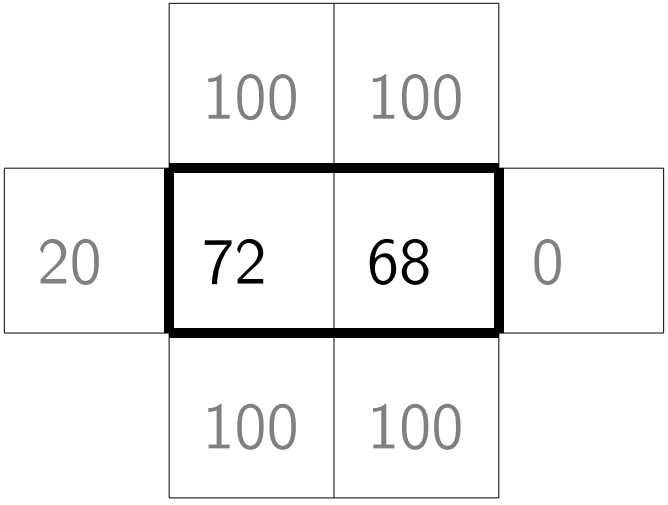
More complicated case.
Monimutkaisempi tilanne.



Form a pair of equations (muodostetaan yhtälöpari):

$$\begin{aligned} x_1 &= (100 + x_2 + 100 + 20)/4, \\ x_2 &= (100 + 0 + 100 + x_1)/4. \end{aligned}$$

More complicated case.
Monimutkaisempi tilanne.



Form a pair of equations (muodostetaan yhtälöpari):

$$\begin{aligned} x_1 &= (100 + x_2 + 100 + 20)/4, \\ x_2 &= (100 + 0 + 100 + x_1)/4. \end{aligned}$$

The solution is (ratkaisu on) $x_1 = 72$, $x_2 = 68$.

How about 3 columns and 4 rows?
Entäpä 3 riviä ja 4 saraketta?

	100	100	100	100
20	x_1	x_4	x_7	x_{10}	0
20	x_2	x_5	x_8	x_{11}	0
20	x_3	x_6	x_9	x_{12}	0
	100	100	100	100	

We get 12 equations.
Saamme 12 yhtälöä.

	100	100	100	100	
20	x_1	x_4	x_7	x_{10}	0
20	x_2	x_5	x_8	x_{11}	0
20	x_3	x_6	x_9	x_{12}	0
	100	100	100	100	

$$
\begin{aligned}
4x_1 - x_2 - x_4 &= 120 \\
-x_1 + 4x_2 - x_3 - x_5 &= 20 \\
-x_2 + 4x_3 - x_6 &= 120 \\
-x_1 + 4x_4 - x_5 - x_7 &= 100 \\
-x_2 - x_4 + 4x_5 - x_6 - x_8 &= 0 \\
-x_3 - x_5 + 4x_6 - x_9 &= 100 \\
-x_4 + 4x_7 - x_8 - x_{10} &= 100 \\
-x_5 - x_7 + 4x_8 - x_9 - x_{11} &= 0 \\
-x_6 - x_8 + 4x_9 - x_{12} &= 100 \\
-x_7 + 4x_{10} - x_{11} &= 100 \\
-x_8 - x_{10} + 4x_{11} - x_{12} &= 0 \\
-x_9 - x_{11} + 4x_{12} &= 100
\end{aligned}
$$

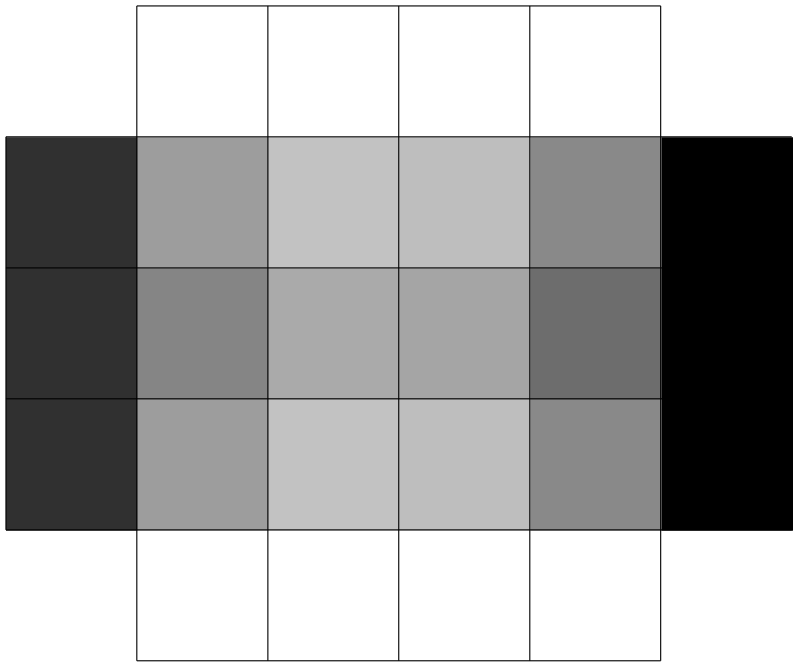
We can use a computer to solve the problem.
Tässä tietokoneen avulla laskettu ratkaisu.

	100	100	100	100	
20	62	76	74	54	0
20	53	67	64	43	0
20	62	76	74	54	0
	100	100	100	100	

$$
\begin{aligned}
4x_1 - x_2 - x_4 &= 120 \\
-x_1 + 4x_2 - x_3 - x_5 &= 20 \\
-x_2 + 4x_3 - x_6 &= 120 \\
-x_1 + 4x_4 - x_5 - x_7 &= 100 \\
-x_2 - x_4 + 4x_5 - x_6 - x_8 &= 0 \\
-x_3 - x_5 + 4x_6 - x_9 &= 100 \\
-x_4 + 4x_7 - x_8 - x_{10} &= 100 \\
-x_5 - x_7 + 4x_8 - x_9 - x_{11} &= 0 \\
-x_6 - x_8 + 4x_9 - x_{12} &= 100 \\
-x_7 + 4x_{10} - x_{11} &= 100 \\
-x_8 - x_{10} + 4x_{11} - x_{12} &= 0 \\
-x_9 - x_{11} + 4x_{12} &= 100
\end{aligned}
$$

Here is the solution as a picture.

Tässä ratkaisu kuvan muodossa.



Now we can solve the previous 6×8 case.
Nyt osaamme ratkaista 6×8 -tapauksenkin.

	100	100	100	100	100	100	100	100	
20	61	77	84	86	86	82	72	51	0
20	46	63	73	76	75	69	56	34	0
20	41	57	67	71	69	62	48	27	0
20	41	57	67	71	69	62	48	27	0
20	46	63	73	76	75	69	56	34	0
20	61	77	84	86	86	82	72	51	0
	100	100	100	100	100	100	100	100	

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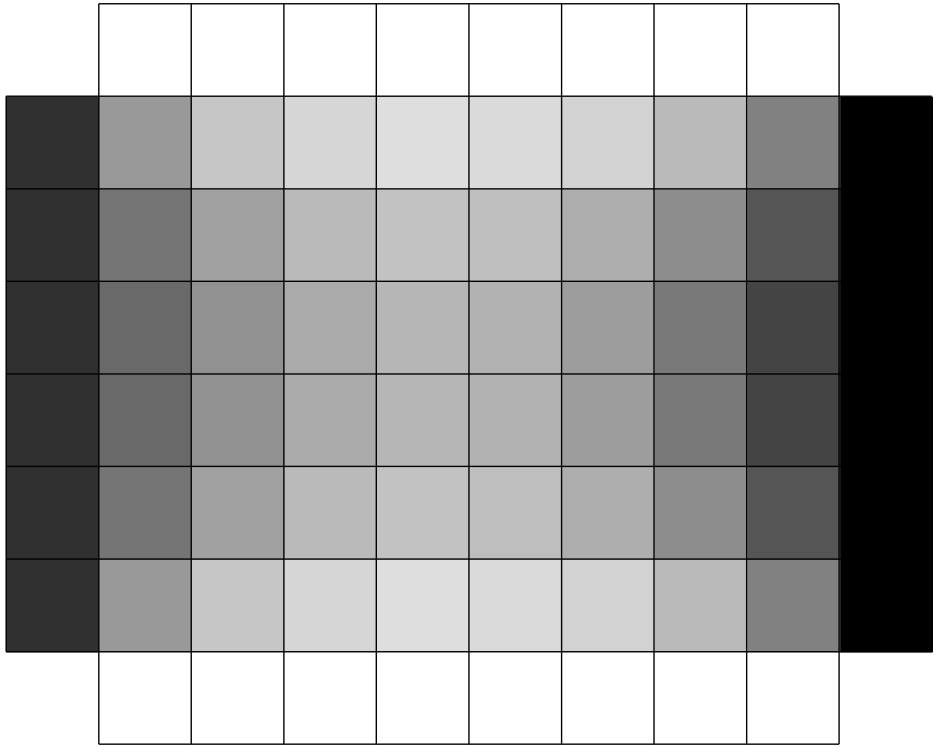
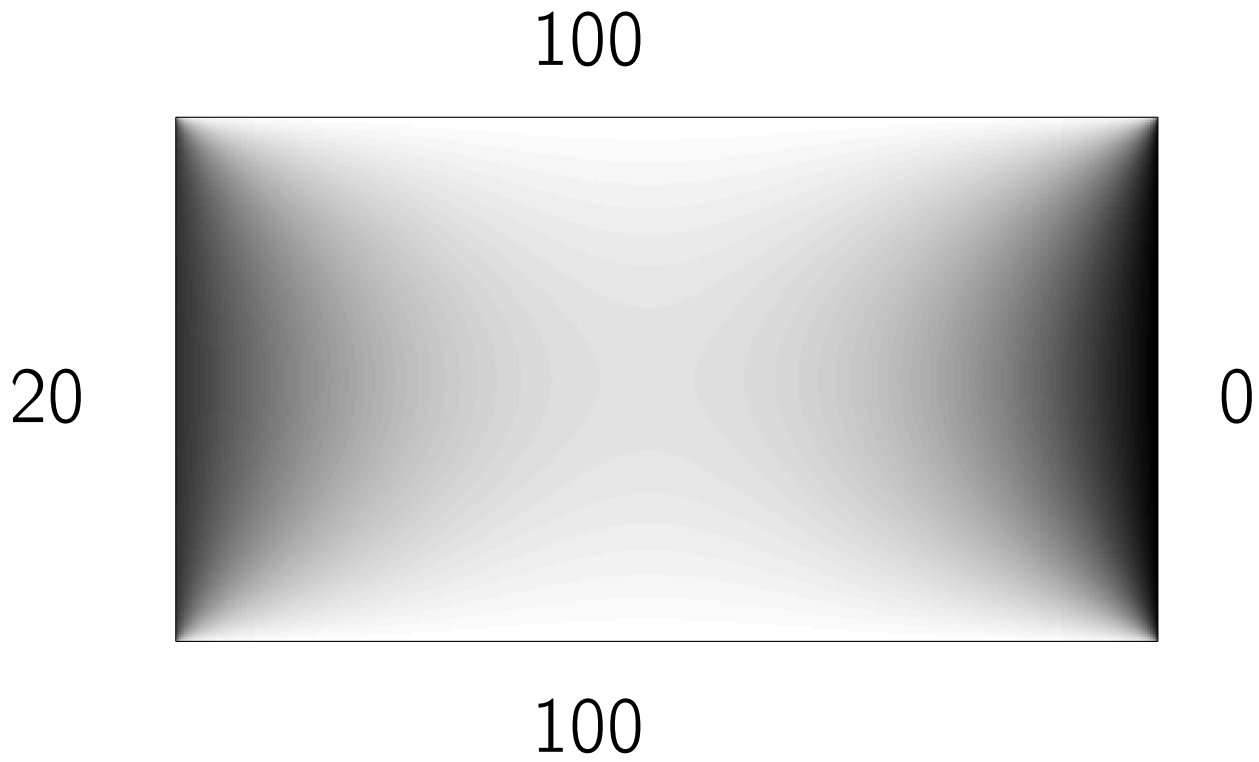


Computers are very powerful!
Tietokoneella on voimaa!

We can solve our problem in the case of 200 rows and 375 columns. This requires solving a system of 75 000 equations!

Valitsemalla 200 riviä ja 375 saraketta saamme 75 000 yhtälön ongelman ratkaistavaksi.

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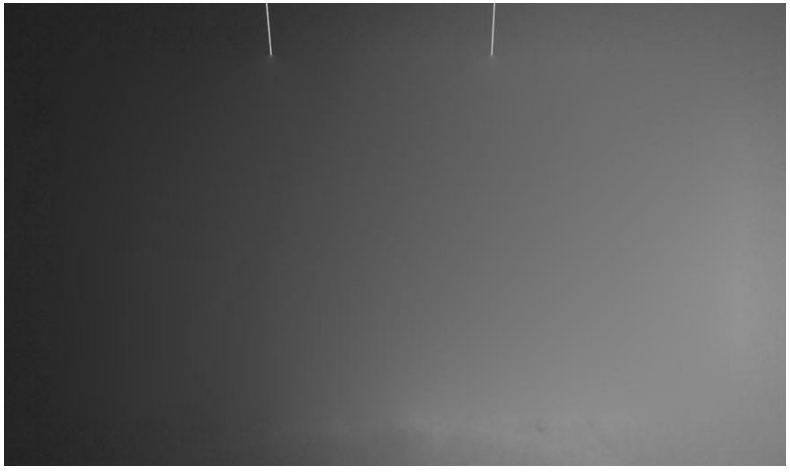
Actually this is the Poisson equation modeling temperature distribution inside a metal plate

$$\begin{aligned} \Delta u &= 0 \text{ in } \Omega, \\ u &= f \text{ on } \partial\Omega. \end{aligned}$$



Siméon Denis Poisson
(1781–1840)

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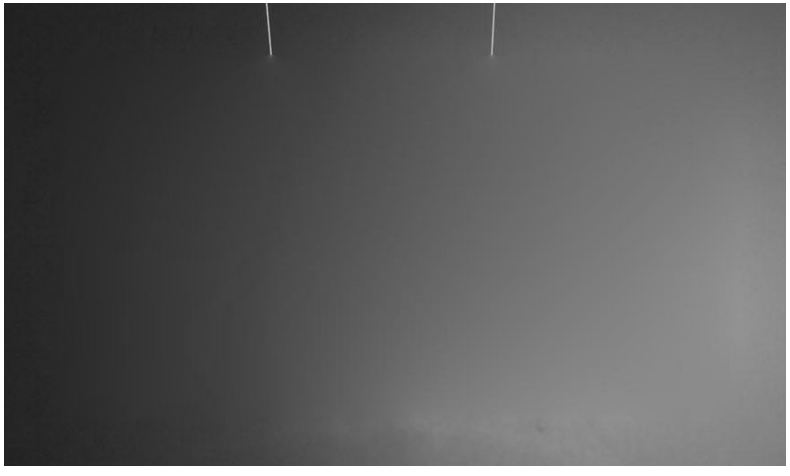

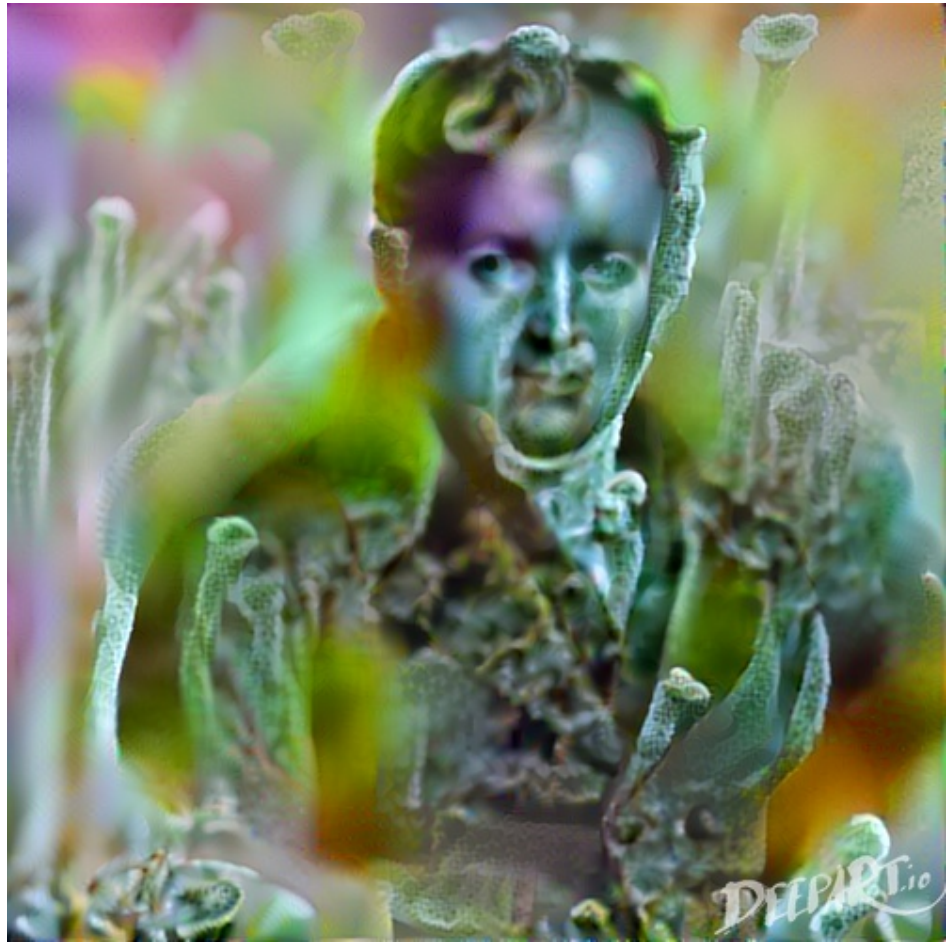

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(1781–1840)