Applications of matrix computations Exercise 1, September 12–16, 2016

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Send your solutions to application.matrixcomputation@gmail.com by Monday 19.09. at 10 AM.
Remember to **include your MATLAB codes, results, images, comments and explanations**, but do try to keep the answers concise; the goal is to show us that you've done and understood the exercises, not to write a thesis. To paraphrase Einstein, make your answers as simple as possible, but no simpler!

1. Define the following matrices in Matlab. For example, the identity matrix of size 2×2 can be given in the form $[[1, 0]; [0, 1]]$.

$$A = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} \pi & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

   Check the sizes of the matrices you created using command `size`.

2. Let us construct some of the matrices in exercise 1 in a quicker way than writing every element explicitly. by using some of the following commands:

   (a) The command `zeros(m,n)` creates an all-zero matrix with m rows and n columns. Construct matrix B using `zeros`.

   (b) The command `ones(m,n)` creates a matrix with all elements equal to one and with m rows and n columns. Construct C by first making a matrix of all ones of the correct size and then writing `C(1,1)=pi`. What happens when you replace `C(1,1)=pi` in the process by `C(1)=pi`?

   (c) Consider (b) in the case when the top right corner has value π instead of top left corner. In other words, replace command `C(1,1)=pi` by `C(1,3)=pi`. What value of `k` in the one-index command `C(k)=pi` gives the same result? Why?

   (d) Use the command `eye` in such a way that the result is D.

   (e) Type `help repmat` into Matlab's command prompt to learn how the command `repmat` works. Use `repmat` to construct E, which consists of four 2×2 matrices that can be created using the command `eye`.

3. Write in Matlab the following commands: $A(:), B(:), \ldots, E(:)$. What happens to the matrices in exercise 1? Explain what you observe.

4. Consider the matrices

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 5 & 1 & 2 & 3 \\ 0 & 5 & 1 & 2 \\ 0 & 0 & 5 & 1 \end{bmatrix}.$$

   (a) Construct matrix G using the command `diag([1,2,3])`. What happens when you enter the command `diag([1,2,3],2)` or `diag([1,2,3],3)`?

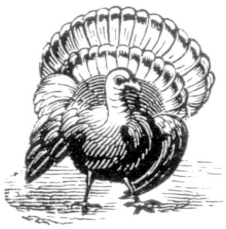
   (b) Use the things you learned in (a) to construct H as a sum of four matrices each created using the command `diag`.

5. You may find this blog post useful for this exercise.

   (a) Plot the function $\sin(2\pi x)$ in the interval $0 \leq x \leq 1$ using the Matlab commands `linspace` and `plot`.

   (b) Plot the three functions $\sin(2\pi x)$ and $\sin(4\pi x)$ and $\sin(6\pi x)$ in three separate plots in the same figure on top of each other. Use the command `subplot(3,1,k)` with appropriate values of `k`.

6. Consider a farm with cows and turkeys. Some garden spiders (*Araneus diadematus*) live at the farm as well. Also, there are rattlesnakes and flies (order *Diptera*). We know that there are 21692 legs, 3606 heads, 6920 wings, 18225 eyes and 50 horns in the farm belonging to these five kinds of animals.


Snake image credit: Andy O'Connor

Anatomical remarks:

- Of course, a spider's body consists of two parts: abdomen and cephalothorax. While the cephalothorax is technically not equal to a head, please work on the assumption of spiders having one head each.
- *Araneus diadematus* spiders have 8 eyes.
- While there are horned orb-web weavers such as *Gasteracantha arcuata* in the family *Araneidae*, the common garden spiders of species *Araneus diadematus* do not have horns.
- Flies do not have horns. They do have antennae, but they are not counted as horns here. Also, flies have two compound eyes and three additional single eyes, making the number of eyes potentially very large depending on interpretation. For simplicity we just say here that a fly has five eyes.
- The kind of rattlesnake considered here has two horns (see picture).

(a) Collect the above information in the form of a 5×5 matrix equation

$$M \begin{bmatrix} r \\ s \\ c \\ t \\ f \end{bmatrix} = b, \qquad b = \begin{bmatrix} 21692 \\ 3606 \\ 6920 \\ 18225 \\ 50 \end{bmatrix},$$

where r is the number of rattlesnakes, s is the number of spiders, c is the number of cows, t is the number of turkeys and f is the number of flies.

(b) Solve the matrix equation in (a) using Matlab and report the numbers of different animals at the farm. Note that there are no mutants on the farm, so your solution should contain only integers.