Äärimmäisten ilmiöiden teoriaa, harj. 4, 14.11.-16

1\. a) Jos X paksuhäntäinen ja $s > 0$, niin

$$\mathbb{E}\left(e^{s(X+Y)}\right) = \mathbb{E}(e^{sX})\,\mathbb{E}(e^{sY}) = \infty$$

b) $$\mathbb{E}\left(e^{sXY}\right) \geq \mathbb{E}\left(e^{sXY} 1(Y \geq \varepsilon)\right)$$

$$\geq \mathbb{P}(Y \geq \varepsilon)\,\mathbb{E}\left(e^{s\varepsilon X}\right) = \infty, \text{ kun } \mathbb{P}(Y \geq \varepsilon) > 0$$

2\. Selvästi $\mathbb{P}(Y_1 + Y_2 > x) \geq \mathbb{P}(Y_2 > x)$, $\forall x$.
Lisäksi

$$\mathbb{P}(Y_1 + Y_2 > x) = \mathbb{P}(Y_1 + Y_2 > x,\ Y_1 \leq M)$$

$$\leq \mathbb{P}(Y_2 > x - M) = (1 + o(1))\,\mathbb{P}(Y_2 > x), \quad x \to \infty.$$

3\. $$\mathbb{E}\left(e^{sX}\right) = \sum_{k=0}^{\infty} (1-c)\,c^k e^{sk}$$

$$= (1-c) \sum_{k=0}^{\infty} \left(e^{s + \log c}\right)^k < \infty, \text{ kun } s + \log c < 0$$

eli kun $s < -\log c$.

$$\mathbb{E}\left(e^{sXY}\right) \geq \mathbb{P}(Y = k)\,\mathbb{E}\left(e^{skX}\right) \text{ on } = \infty,$$

kun $sk \geq -\log c$ eli kun $k \geq -\frac{\log c}{s}$.

4\. $$\mathbb{P}(M_n > x) = 1 - F(x)^n = (1 - F(x))\left(1 + F(x) + \dots + F(x)^{n-1}\right)$$

$$= (1 + o(1))\, n \bar{F}(x), \quad x \to \infty,$$

$$\lim_{t \to \infty} \frac{\mathbb{P}(M_n \geq tx)}{\mathbb{P}(M_n \geq t)} = \lim_{t \to \infty} \frac{\bar{F}(tx)}{\bar{F}(t)} = x^{-\alpha}.$$

5. $\mathbb{P}(\xi_1 + \xi_2 > x, \xi_2 + \xi_3 > x) \geq \mathbb{P}(\xi_2 > x) = \bar{F}(x)$,

$\mathbb{P}(\xi_1 + \xi_2 > x, \xi_2 + \xi_3 > x)$

$\leq \mathbb{P}(\xi_2 > (1-\varepsilon)x) + \mathbb{P}(\xi_2 \leq (1-\varepsilon)x, \xi_1 > \varepsilon x, \xi_3 > \varepsilon x)$

$\leq \bar{F}((1-\varepsilon)x) + \bar{F}(\varepsilon x)^2$

$= (1+o(1))(1-\varepsilon)^{-\alpha}\bar{F}(x) + (1+o(1))\varepsilon^{-2\alpha}\bar{F}(x)$

$= (1+o(1))(1-\varepsilon)^{-\alpha}\bar{F}(x)$

Så

$$\limsup_{x\to\infty} \frac{\mathbb{P}(\xi_1+\xi_2>x, \xi_2+\xi_3>x)}{\bar{F}(x)} \leq (1-\varepsilon)^{-\alpha},$$

$$\Rightarrow \limsup_{x\to\infty} \frac{\mathbb{P}(\xi_1+\xi_2>x, \xi_2+\xi_3>x)}{\bar{F}(x)} \leq 1.$$