

6)

$$a) f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$f_X(x) = \int f_{X,Y}(x,y) dy = \frac{24}{11} (x+1) \cdot \mathbb{1}\{0 < x < 1\} \cdot \int_x^1 y dy$$

$$= \frac{24}{11 \cdot 2} (x+1) \cdot (1-x^2) \mathbb{1}\{0 < x < 1\}$$

$$\therefore f_{Y|X}(y|x) = \frac{\frac{24}{11} (x+1)}{\frac{24}{11 \cdot 2} (x+1)} \cdot \frac{y}{1-x^2} \mathbb{1}\{0 < x < y < 1\}$$

$$= \frac{2y}{1-x^2} \mathbb{1}\{0 < x < y < 1\}$$

$$b) E(Y|X=x) = \int f_{Y|X}(y|x) \cdot y dy$$

$$= \frac{2}{1-x^2} \mathbb{1}\{0 < x < 1\} \int_x^1 y^2 dy$$

$$= \frac{2}{3(1-x^2)} \cdot \mathbb{1}\{0 < x < 1\} \cdot (1-x^3) = \frac{2}{3} \frac{1+x+x^2}{1+x} \cdot \frac{1-x}{1-x} \cdot \mathbb{1}\{0 < x < 1\}$$

$$= \frac{2}{3} \frac{x(x+1)+1}{x+1} \cdot \mathbb{1}\{0 < x < 1\} = \frac{2}{3} \left(x + \frac{1}{x+1}\right) \mathbb{1}\{0 < x < 1\}$$

$$\therefore E(Y|X) = \frac{2}{3} \left(X + \frac{1}{X+1}\right) \quad \left( \begin{array}{l} \text{niilä } \mathbb{1}\{0 < X < 1\} = 1 \\ \text{nikillä } 1 \end{array} \right)$$

c) Paras lin. ennuste on

$$EY + \frac{\text{cov}(X,Y)}{\text{var } X} \cdot (X - EX)$$

$$EY = \int y f_{X,Y}(x,y) dx dy = \int dx \int \frac{24}{11} (x+1) \cdot y^2 dy$$

$$= \frac{24}{3 \cdot 11} \int_0^1 (x+1) (1-x^3) dx = \frac{24}{3 \cdot 11} \int_0^1 (1+x-x^3-x^4) dx$$