Matematiikan ja tilastotieteen laitos
Reaalianalyysi II
Harjoitus 8
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In the following $\mu : \mathcal{M} \to \mathbb{R} \cup \{-\infty, \infty\}$ is a signed measure, where $\mathcal{M}$ is a σ-algebra in a set X.

1. Show that if $A, B \in \mathcal{M}$, $A \subset B$ and $\mu(A) = \infty$, then $\mu(B) = \infty$.

2. Show that the measures μ^+ and μ^- in the Jordan decomposition are unique.

3. Show that if $\nu : \mathcal{M} \to [0, \infty]$ is a measure, then $\mu << \nu$, if and only if $V(\mu, .) << \nu$.

In the following $\mu : \mathcal{M} \to [0, \infty]$ is a measure. The set $A \in \mathcal{M}$ is an atom of μ if $\mu(A) > 0$ and for all $B \in \mathcal{M}, B \subset A$, either $\mu(B) = \mu(A)$ or $\mu(B) = 0$.

4. What are the atoms of the Lebesgue measure and the counting measure?

5. Show that the atoms of every Radon measure μ in $\mathbb{R}^n$ are almost singletons, that is, for every atom A of μ there is $a \in A$ such that $\mu(A \setminus \{a\}) = 0$.

6. Show that if μ has no atoms and $0 < t < \mu(X) < \infty$, then there is $A \in \mathcal{M}$ such that $\mu(A) = t$.