Partial Differential Equations
University of Helsinki, Department of Mathematics and Statistics
Fall 2015
Home assignment 7

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Problem set I

In the next problems we fill in the missing details of the Perron's construction of the Dirichlet problem.

1. Study the Corollary 2.5.1 (Harnack's Inequality) in DiBenedetto and fill in the missing details in its proof.

2. Assume that (v_n) is an increasing sequence of non–negative harmonic functions in the ball $B_r(x_0)$. Assume that there exists a finite limit $\lim_{n\to\infty} v_n(x_0)$. Show that the sequence $v_n(x)$ converges to a finite limit at every point $x \in B_r(x_0)$.

3. Under the assumptions of the previous problem, show that the sequence (v_n) is equicontinuous in compact subsets of $B_r(x_0)$. **Hint**: Apply Poisson's formula in a suitable ball $B_\rho(x_0)$, $\rho < r$, and estimate derivatives using this.

4. Shoe that there is a limit function
$$u(x) = \lim_{n\to\infty} v_n(x)$$
which is continuous in the ball $B_r(x_0)$.

5. Prove that u above is harmonic in $B_r(x_0)$.

Problem set II

For the next problems read sections 5.10 ja 5.11 of DiBenedetto.

1. How is defined the initial value at $t = 0$ of a solution $u(x,t)$ with respect to the L^2–norm?

2. Assume $\Omega \subset \mathbb{R}^d$ is bounded, and $u \in C^{2,1}(\Omega \times \mathbb{R}_+)$ solves the initial boundary value problem
$$\partial_t u - \Delta u = 0 \text{ in } \Omega \times \mathbb{R}_+,\ u|_{t=0} = u_0,\ u|_{\partial\Omega\times\mathbb{R}_+} = 0,$$
where $u_0 \in C_0^1(\Omega)$. Prove that for all $t > 0$ we have the *Energy Identity*
$$\frac{1}{2}\|u(\cdot,t)\|_{L^2(\Omega)}^2 - \frac{1}{2}\|u_0\|_{L^2(\Omega)}^2 + \int_0^t \int_\Omega |\nabla_x u|^2 \, dx \, dt = 0.$$

3. Assume that above $u_0 = 0$.What can you say about the solution u?