Partial Differential Equations
University of Helsinki, Department of Mathematics and Statistics
Fall 2015
Home assignment 6

Return by: ma 12.10.2015 klo 19.30
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Problem set I

1. Prove that a continuous function v is subharmonic in an open set Ω if and only if for every bounded open subset $\Omega' \subset \Omega$ and for every function u harmonic in Ω' and continuous up to the boundary, for which $u|_{\partial\Omega'} = v_{\partial\Omega'}$, we have $v \leq u$ in Ω'.

2. Assume $u \in C^2(\Omega) \cap C(\overline{\Omega})$ solves the *inhomogenous* Dirichlet–problem
$$\Delta u = -1 \quad \text{in } \Omega,\ u|_{\partial\Omega} = 0.$$
Prove that
$$u(x) \geq \frac{1}{2d} \inf_{y \in \partial\Omega} |x-y|^2 \quad \text{for all } x \in \Omega.$$

Problem set II

Next we continue our quest of proving the Ascoli–Arzéla Theorem.

3. Prove that if a subset K of a metric spaces is compact, then every infinite subset K has a limit point in K.

4. We define, that a subset K of a metric space is *totally bounded*, if for all $\varepsilon > 0$ there exists a finite open cover of K with open balls of radius $\varepsilon > 0$. Prove that if every infinite subset K has a limit point in K, then it is totally bounded.

Problem set III

5. Assume that $f \in C_0^2(\mathbb{R}^d)$ and let Φ be the fundamental solution of the Laplace–operator in $\mathbb{R}^d$. Define
$$u(x) = \int_{\mathbb{R}^d} \Phi(x-y) f(y)\, dy, \quad x \in \mathbb{R}^d. \tag{1}$$
Show that $f \in C^2(\mathbb{R}^d)$.

6. Let f be as in the previous exercise, and define also u by (**??**). Prove, that
$$\Delta u = f \quad \text{in } \mathbb{R}^d.$$

Problem set IV

Read pages 135-140 of DiBenedetto.

7. Assume $t > 0$. Show that
$$\int_{\mathbb{R}^d} e^{-|x|^2/4t}\, dx = (4\pi t)^{d/2}.$$

8. Explain, how the Maximum Principle for the solutions of the heat equation differs from the Maximum Principle of harmonic functions.