Partial Differential Equations
University of Helsinki, Department of Mathematics and Statistics
Fall 2015
Home assignment 5

Return by: Monday 05.10.2015 klo 19.30
Return corrections by: Monday 12.10.2015 klo 19.30

Problem set I

1. Study the Dirichlet Problem

$$\Delta u = 0 \quad \text{in } B_R(0)$$

$$u|_{\partial B_R} = \begin{cases} 0, & \text{jos } x_d < 0 \\ 1, & \text{jos } x_d \geq 0 \end{cases}.$$

   Hint: Eventhough the boundary value is not continuous, you may still attempt to use Poisson integral. Study the boundary behaviour near discontinuities separately.

2. Can you come up with a way to construct the Dirichlet Green's function for the Laplacian in the set

$$B_R(0)^+ = B_R(0) \cap \mathbb{R}^d_+?$$

Problem set II

3. Assume that $u \in C^2(\Omega)$, $\Omega \subset \mathbb{R}^d$ open, solves the Helmholtz equation

$$\Delta u - u = 0.$$

   Show that u can't have a positive local maxima, nor a negative local minima.

4. Assume that u is harmonic. Prove that $|\nabla u|^2$ is subharmonic.

Problem set III

Read the section 2.6, *The Dirichlet Problem*, of DiBenedetto.

5. Explain in your own words, what the *exterior sphere condition* defined on page 55 means.
6. Give examles of domains for which this condition is true, and also for which it fails.

Problem set IV

We prove the solvability of the Dirichlet problem using the so called Perron's method. In the proof we need the Ascoli–Arzela Theorem. We will not prove this in the lectures. However, on this week's and next week's homeworks we will cover the main ideas of the proof. We start with a central concept, namely the *equicontinuity*.

7. Read for example from the end of Appendix C in Evans (2nd edition) how the equicontinuity of a sequence of functions $\{f_k\}$ is defined, and try to explain this in your own words. Can you give an example of a sequence which is **not** equicontinuous

8. Let $\Omega \subset \mathbb{R}^d$ be a domain. Assume that $f_k \in C^1(\Omega)$, $k \in \mathbb{N}$, and that the derivatives $\partial_i f_k$ are uniformly bounded, i.e there exists a constant M such that

$$|\partial_i f_k(x)| \leq M \quad \text{kaikilla } i \in \{1, \dots, d\},\ k \in \mathbb{N} \text{ ja } x \in \Omega.$$

   Is the sequence f_k equicontinuous?