Partial Differential Equations
University of Helsinki, Department of Mathematics and Statistics
Fall 2015
Home assignment 2

Return by: Monday 14.9.2015 7.30 pm
Return corrections by: Monday 21.9.2015 klo 7.30 pm

We say that the function $h : \mathbb{R}^n \to \mathbb{R}$ determines a C^k–set U, $k \geq 1$, if the following conditions hold:

- $h \in C^k(\mathbb{R}^n)$.
- We have $U = \{x \in \mathbb{R}^n;\ h(x) < 0\}$ and $\mathbb{R}^n \setminus \overline{U} = \{x \in \mathbb{R}^n;\ h(x) > 0\}$
- For the boundary we have $\partial U = \{x \in \mathbb{R}^n;\ h(x) = 0\}$.
- In addition the following must hold: If $h(x) = 0$, then $\nabla h(x) \neq 0$.

Problem set I

1. If h determines a C^k–set U, what is *the exterior unit normal*, i.e the normal vector to the boundary having length $= 1$, and pointing *away* from U?
2. Give an example of a C^1–set, and its exterior unit normal.

Problem set II

3. Does the function $h(x, y) = y^2 - x^4$ determine a C^1–set of the plane? Illustrate this by drawing a picture.
4. Does the function $h(x, y) = xy - 1$ determine a C^1–set? Is this set bounded, or connected? Illustrate also this by drawing a picture.

Problem set III

Read sections 1.2 ja 1.3 of DiBenedetto

5. Explain the difference between the Dirichlet–problem, Neumann–problem and Cauchy–problem for the laplace–operator
6. Explain, why the example due to Hadamard in section 1.3 shows that the Cauchy–problem for the Laplace–operator is ill–posed.

Problem set IV

7. How the argument of Lemma 1.1 in DiBenedetto has to be changed so that it also proves the claim concerning the Neumann–problem?
8. Assume, that a *radial* function $f(x) = V(|x|)$ satisfies $\Delta f = 0$ in $R^n \setminus 0$. Prove, that the function $V(r)$ of one variable solves the ordinary differential equation

$$V'' + \frac{n-1}{r} V' = 0. \tag{1}$$

9. Solve the above equation (1) in dimensions $n = 2$ ja $n = 3$.