Partial Differential Equations
University of Helsinki, Department of Mathematics and Statistics
Fall 2015
Home assignment 1

Return by: Monday 7.9.2015 7.30 pm
Return corrections by: Monday 14.9.2015 klo 7.30 pm

The first two problem sets are just for recalling some basic facts on differential equations and calculus. The two last ones act as a gentle introduction to concepts in next week lectures

Problem set I

You may consult a text book on calculus of several variables.

1. Evaluate the gradients of the following two functions:
$$f(x,y) = e^{x^2+y^2}\sin(x-y),\ g(x,y,z) = \frac{x^2-y^2}{z^2+e^{x+y}}.$$
2. Explain what is the directional derivative of the function f to the direction of a vector α, $|\alpha| = 1$. What does the gradient of a function tell of its growth?

Problem set II

You may consult your favourite text book on differential equations.

3. Assume that y solves the initial value problem
$$y' = \frac{y^3-y}{1+x^4y^2},\ y(0) = 1/2.$$
Show that on its domain of definition $0 < y(x) < 1$.
4. Solve the initial value problem
$$y' = Ay,\ y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
when
$$A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}.$$

Problem set III

Read the introduction of Chapter 2 in DiBenedetto, or alternatively the introduction 2.2.1 a) in Evans.

5. Formulate precisely the claim that *Laplace operator is translation invariant*, and prove it.
6. Formulate precisely the claim that *Laplace operator is rotation invariant*, and prove it.

Problem set IV

7. Let $u \in C^2(E)$, where $E \subset \mathbb{R}^n$ is open. Assume, that $\Delta u(x) > 0$ for all $x \in E$. Can the function u have a strict local maxima in E? What about a strict local minima?
8. Assume that $u \in C^2(\mathbb{R}^n \times (0,\infty))$ solves the heat equation,
$$u_t(x,t) - \Delta u(x,t) = 0, \quad (x,t) \in \mathbb{R}^n \times (0,\infty).$$
Prove that for all $\lambda \in \mathbb{R}$ the function $u_\lambda(x,t) = u(\lambda x, \lambda^2 t)$ also solves the heat equation