Partial Differential Equations
University of Helsinki, Department of Mathematics and Statistics
Fall 2015
Home assignment 13

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Problem set I

This time only one set of questions, and five problems. This is a practice exam :)

1. Solve the initial value problem

$$\frac{\partial^2 u(x,t)}{\partial t^2} - c^2 \frac{\partial^2 u(x,t)}{\partial x^2} = x^2, \quad x \in \mathbb{R},\ t > 0,$$
$$u(x,0) = x, \quad \frac{\partial u(x,0)}{\partial t} = 0, \quad x \in \mathbb{R}.$$

2. Find all solution of the equation

$$\frac{1}{h}\frac{\partial u(x,t)}{\partial t} - i\frac{\partial^2 u(x,t)}{\partial x^2} = 0, 0 < x < \pi,\ t > 0,$$

which are of the form $u(x,t) = X(x)T(t)$ for which

$$u(0,t) = u(\pi,t) = 0,\ t > 0.$$

Here h is a positive constant.

3. Let $\Omega = (0,l) \times \mathbb{R}_+$, $v \in C^2(\overline{\Omega})$ and assume that for all $(x,t) \in \Omega$ it holds

$$\frac{\partial v(x,t)}{\partial t} - \frac{\partial^2 v(x,t)}{\partial x^2} = 0,$$

and that for the initial values we have

$$v(x,0) = 0, \quad 0 < x < l,$$

and

$$v(0,t) = v(l,t) = 0, \quad t > 0.$$

Prove that $v = 0$.

4. Solve the initial boundary value problem

$$\frac{\partial^2 u(x,t)}{\partial t^2} - \frac{\partial^2 u(x,t)}{\partial x^2} = 0, \quad 0 < x < \pi,\ t > 0,$$
$$u(x,0) = \sin x + \sin^2 x, \quad \frac{\partial u(x,0)}{\partial t} = 0, \quad 0 < x < \pi.$$
$$u(0,t) = u(\pi,t) = 0, \quad t > 0.$$

The following problem is only included in the final exam.

5. Construct a (Dirichlet) Green's function for the Laplace–operaattor Δ in the unit ball $B(0,1) = \{x \in \mathbb{R}^3;\ \|x\| < 1\}$.