

Modaalilogiikka, harjoitus 6 (14.10.2015)

Ratkaisuja

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1. (a) (1) $\Box\Box A \rightarrow \Box A$ (T)
 (2) $\Box A \rightarrow \Box\Box A$ (4)
 (3) $\Box\Box A \leftrightarrow \Box A$ (PL, 1, 2)
- (b) (1) $\Box\Diamond A \rightarrow \Diamond A$ (T)
 (2) $\Diamond A \rightarrow \Box\Diamond A$ (5)
 (3) $\Box\Diamond A \leftrightarrow \Diamond A$ (PL, 1, 2)
- (c) (1) $\Diamond\Box A \rightarrow \Box A$ ($5\Diamond$)
 (2) $\Box A \rightarrow \Diamond\Box A$ ($T\Diamond$)
 (3) $\Diamond\Box A \leftrightarrow \Box A$ (PL, 1, 2)
- (d) (1) $\Diamond\Diamond A \rightarrow \Diamond A$ ($4\Diamond$)
 (2) $\Diamond A \rightarrow \Diamond\Diamond A$ ($T\Diamond$)
 (3) $\Diamond\Diamond A \leftrightarrow \Diamond A$ (PL, 1, 2)

2. (a)

$$\begin{aligned} \Diamond \neg \Diamond \Box (p_1 \rightarrow p_2) &\equiv_{\mathbf{S5}} \neg \Box \neg \neg \Diamond \Box (p_1 \rightarrow p_2) \\ &\equiv_{\mathbf{S5}} \neg \Box \Diamond \Box (p_1 \rightarrow p_2) \\ &\equiv_{\mathbf{S5}} \neg \Diamond \Box (p_1 \rightarrow p_2) \\ &\equiv_{\mathbf{S5}} \neg \Box (p_1 \rightarrow p_2) \end{aligned}$$

(b)

$$\begin{aligned}
\neg \diamond \neg \square \diamond \neg p_1 &\equiv_{\mathbf{S5}} \neg \neg \square \neg \neg \square \diamond \neg p_1 \\
&\equiv_{\mathbf{S5}} \square \square \diamond \neg p_1 \\
&\equiv_{\mathbf{S5}} \square \diamond \neg p_1 \\
&\equiv_{\mathbf{S5}} \diamond \neg p_1 \\
&\equiv_{\mathbf{S5}} \neg \square \neg \neg p_1 \\
&\equiv_{\mathbf{S5}} \neg \square p_1
\end{aligned}$$

3. (a)
- (1) $A \rightarrow (B \rightarrow C)$ (oletus)
 - (2) $\square A \rightarrow \square(B \rightarrow C)$ (RM, 1)
 - (3) $\square(B \rightarrow C) \rightarrow (\square B \rightarrow \square C)$ (K)
 - (4) $\square A \rightarrow (\square B \rightarrow \square C)$ (PL, 2, 3)

(Tämä tehtävä oli erehdyksessä toiseen kertaan ratkaistavana. Pa-
hoittelen.)

- (b)
- (1) $\square(A \wedge B) \rightarrow (\square A \wedge \square B)$ (harj. 3, teht. 1)
 - (2) $A \rightarrow (B \rightarrow (A \wedge B))$ (PL)
 - (3) $\square A \rightarrow (\square B \rightarrow \square(A \wedge B))$ (a-kohta, 2)
 - (4) $\square(A \wedge B) \leftrightarrow (\square A \wedge \square B)$ (PL, 1, 3)

4.

$$\begin{aligned}
\square(p_1 \wedge \neg \square(\diamond p_1 \wedge \neg p_2)) &\equiv_{\mathbf{S5}} \square p_1 \wedge \square \neg \square(\diamond p_1 \wedge \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge \neg \diamond \neg \neg \square(\diamond p_1 \wedge \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge \neg \diamond \square(\diamond p_1 \wedge \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge \neg \square(\diamond p_1 \wedge \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge \neg(\square \diamond p_1 \wedge \square \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge \neg(\diamond p_1 \wedge \square \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge (\neg \diamond p_1 \vee \neg \square \neg p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge (\neg \diamond p_1 \vee \diamond p_2) \\
&\equiv_{\mathbf{S5}} (\square p_1 \wedge \neg \diamond p_1) \vee (\square p_1 \wedge \diamond p_2) \\
&\equiv_{\mathbf{S5}} \square p_1 \wedge \diamond p_2
\end{aligned}$$

Kaksi viimeistä sievennysaskelta perustuivat skeemaan (D) ja proposi-
tiologiikkaan.

5. (a)
- (1) $A \rightarrow (A \vee B)$ (taut)
 - (2) $\Box A \rightarrow \Box(A \vee B)$ (RM, 1)
 - (3) $B \rightarrow (A \vee B)$ (taut)
 - (4) $\Box B \rightarrow \Box(A \vee B)$ (RM, 3)
 - (5) $(\Box A \vee \Box B) \rightarrow \Box(A \vee B)$ (PL, 2, 4)
- (b)
- (1) $(\Box A \vee \Box \Box B) \rightarrow \Box(A \vee \Box B)$ (a)
 - (2) $\Box B \rightarrow \Box \Box B$ (4)
 - (3) $(\Box A \vee \Box B) \rightarrow \Box(A \vee \Box B)$ (PL, 1, 2)
 - (4) $(A \vee \Box B) \rightarrow (\neg \Box B \rightarrow A)$ (PL)
 - (5) $\Box(A \vee \Box B) \rightarrow \Box(\neg \Box B \rightarrow A)$ (RM, 4)
 - (6) $\Box(\neg \Box B \rightarrow A) \rightarrow (\Box \neg \Box B \rightarrow \Box A)$ (K)
 - (7) $\Box(A \vee \Box B) \rightarrow (\Box A \vee \Diamond \Box B)$ (PL, 5, 6)
 - (8) $\Diamond \Box B \rightarrow \Box B$ (5 \Diamond)
 - (9) $\Box(A \vee \Box B) \leftrightarrow (\Box A \vee \Box B)$ (PL, 3, 7, 8)

6.

$$\begin{aligned}
\Diamond(p_1 \wedge \neg \Box(\neg p_1 \vee \Diamond p_2)) &\equiv_{\mathbf{S5}} \Diamond(p_1 \wedge \neg \Box \neg(\neg \neg p_1 \wedge \neg \Diamond p_2)) \\
&\equiv_{\mathbf{S5}} \Diamond(p_1 \wedge \Diamond(p_1 \wedge \neg \Diamond p_2)) \\
&\equiv_{\mathbf{S5}} \Diamond p_1 \wedge \Diamond(p_1 \wedge \neg \Diamond p_2) \\
&\equiv_{\mathbf{S5}} \Diamond p_1 \wedge \Diamond(p_1 \wedge \Box \neg p_2) \\
&\equiv_{\mathbf{S5}} \Diamond p_1 \wedge \Diamond p_1 \wedge \Box \neg p_2 \\
&\equiv_{\mathbf{S5}} \Diamond p_1 \wedge \Box \neg p_2
\end{aligned}$$