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MATHEMATICS OF INFECTIOUS DISEASES

Syllabus of the fall 2015 book reading course based on Diekmann, Heesterbeek & Britton: Mathematical tools for understanding infectious disease dynamics (Princeton University Press 2012). The syllabus is updated as the course proceeds.

6 – 21 September (18 September cancelled)

Five lectures of introduction, covers sections 1.1-1.2 and other material.
1.2.4 Interlude on terminology and notation – *read it yourself*

1.3 THE FINAL SIZE

25 September

Quick test: first five lectures AND sections 1.1, 1.2.1-1.2.2, 1.2.4 of the book

[1] 1.3.1 The standard final-size equation (exercises 1.19-1.20; ADD implicit function theorem: Wikipedia is good but reduce to 2D; show the numerical solution of the final-size equation as a function of R_0, this takes care of exercises 3.13-3.14, Fig 16.14)
[2] 1.3.2 Derivation of the standard final-size equation (exercise 1.22 partly done already; exercises 1.23-1.24)
[3] 1.3.3 The final size of epidemics within herds, part 1: pp. 21-22 (exercises 1.25-1.28)

28 September

[4] 1.3.3 The final size of epidemics within herds, part 2: pp. 23-24 (exercises 1.29-1.33)
[5] 1.3.4 The final size in a finite population: exercise 1.34; ADD: carry out the numerics and show the final-size distribution (1.35 optional)
Eva presents section 1.2.3 (initial growth in real time)

2 October

2.1 DIFFERENCES IN INFECTIVITY

[6] part 1 of 2.1: pp. 33-35 (exercises 2.1-2.4)
[7] part 2 of 2.1: bottom of p 36 ("An attractive class of sub-models...") – halfway p 38 (exercises 2.7-2.9 and also add 2.5, but skip 2.6 [later])
[8] part 3 of 2.1: halfway p 38 ("In concluding this section...")– halfway p 39 (exercises 2.10-2.11)

5 October

[E] EXERCISES done by all and discussed as in an ordinary exercise class: 1.39-1.44 (take 1.42-iii with a pinch of salt!) plus replace 1.37 with this: Obtain the final size of the epidemic as a function of *v*, the fraction of the population vaccinated before the outbreak. I also recommend revisiting 1.26(iii)-(v)

9 October

2.2 DIFFERENCES IN INFECTIVITY AND SUSCEPTIBILITY – *Eva presents this*

2.3 THE PITFALL OF OVERLOOKING DEPENDENCE
[9] The whole section (exercise 2.15). Before the exercise, work out the probability distribution of the number of infected neighbours [p(i)] for arbitrary parameters δ and γ; then take the analytically tractable special case $\delta = \gamma$. ADD exercise 3.3 from Chapter 3 (same issue).

3.1 THE PROTOTYPE STOCHASTIC EPIDEMIC MODEL
3.2 TWO SPECIAL CASES
Pp. 45-50 is mostly covered already, but do <u>read it yourself because</u> new terms (used in stochastic models) are introduced

3.3 INITIAL PHASE OF THE STOCHASTIC EPIDEMIC IN A LARGE POPULATION
Much of pp. 51-58 is a review of previous material, but do read it yourself. Exercise 3.10 is important!

[10] 3.3.1 Continuous-time branching processes; and
3.3.2 Approximation of the initial phase of the epidemic
Present only the distribution of the final size (including exercises 3.8-3.9 [compare with exercise 1.41] and exercise 3.12)

12 October

3.4 APPROXIMATION OF THE MAIN PART OF THE EPIDEMIC – *Read it yourself.*
3.5 APPROXIMATION OF THE FINAL SIZE: THE SELLKE CONSTRUCTION
3.5.1 Law of large numbers limit: a heuristic argument – *review of previous material, read it yourself*
3.5.2-3.5.4 The Sellke construction – *Eva presents this*
3.6 THE DURATION OF THE EPIDEMIC – *Read it, prepare for a brief discussion of Fig 3.8.*

16 October

Quick test

4.1 REPEATED OUTBREAKS VERSUS PERSISTENCE – *Read it yourself.*
[11] **4.2 FLUCTUATIONS AROUND THE ENDEMIC STEADY STATE**: pp. 75-77 (exercises 4.1-4.5)
[12] **4.3 VACCINATION** (exercises 4.19-4.22; ADD modelling the possibilities described in the last paragraph)
[13] **4.4 REGULATION OF HOST POPULATIONS** (exercises 4.23-4.27)

---------- TEACHING BREAK ----------

26 October

Linear stability analysis (*Eva*)

30 October

Quick test on linear stability analysis (find the equilibria and establish their asymptotic stability in a concrete problem)

[E] EXERCISES done by all and discussed as in an ordinary exercise class: 4.6, 4.8, 4.10, 4.13, 4.14, 4.16 (NB 4.15 already done; 4.17 left for later)
[14] **4.6 MARKOV CHAINS: MODELS OF INFECTION IN THE ICU** (exercises 4.28-4.32)

4.7 TIME TO EXTINCTION AND CRITICAL COMMUNITY SIZE – *We omit this chapter for lack of time*

2 – 6 November

4.5 TOOLS FOR EVOLUTIONARY CONTEMPLATIONS – *Eva presents this and other evolutionary models*
5. INFERENCE – *Read sections 5.1-5.3, we omit the rest*

9 November

Quick test on evolutionary models (lectures of 2-6 November) and inference (sections 5.1-5.3, independent study)

6. THE CONCEPT OF STATE – *read this short chapter yourself. The concepts introduced here are essential for the remainder of the course!*
[15] **7.1 THE DEFINITION OF R_0** (exercises 7.1, 7.2, 7.5, 7.7). Use the example of section 2.2 to illustrate (this is also the basis of 7.7).
7.2 NEXT-GENERATION MATRIX FOR COMPARTMENTAL SYSTEMS
[16] Pp. 166-170 (exercises 7.8-7.10 and recall material from section 2.1 as needed, including exercise 2.6)
(with extra discussion of the next generation matrix, including exercise 2.6)

13 November

[E] EXERCISES done by all and discussed as in an ordinary exercise class: 7.11-7.14

[17] **7.3 GENERAL h-STATE** and
7.4 CONDITIONS THAT SIMPLIFY THE COMPUTATION OF R_0 (exercises 7.17-7.19; section 7.4.3 is optional)

16 November

Quick test on sections 7.1-7.2

[18] **7.5 SUB-MODELS FOR THE KERNEL** (exercise 7.22, ADD one concrete example of your choice, where each part of the model is derived from specific assumptions about contact, infectivity, etc.)
7.6 SENSITIVITY ANALYSIS OF R_0 – *read it yourself, optional.*
[19] **7.8 PAIR FORMATION MODELS**

20 November

7.9 INVASION UNDER PERIODIC ENVIRONMENTAL CONDITIONS – *Eva presents this*

23 November

Quick test on sections 7.3-7.5

[20] **7.10 TARGETED CONTROL**
8.1 THE PROBABILITY OF A MAJOR OUTBREAK
[21] 8.1.1 A simplistic model for an STD in a heterosexual community
[22] 8.1.2 Partially vaccinated populations
8.2 THE INTRINSIC GROWTH RATE – *we omit this section*

27 November *exercise class, Eva not here*

7.7 EXTENDED EXAMPLE: TWO DISEASES
[E] EXERCISES done by all and discussed as in an ordinary exercise class: 7.23-7.30 (7.31-7.32 omitted); 7.33-7.34 (7.35-7.38 omitted)

30 November

Quick test on sections 7.8-7.10 and 8.1

[23] **8.3 A BRIEF LOOK AT FINAL SIZE AND ENDEMIC LEVEL**
[24] **8.4 SIMPLIFICATIONS UNDER SEPARABLE MIXING**
[25] **9.1 DEMOGRAPHY** and
9.2 CONTACTS and
9.3 THE NEXT-GENERATION OPERATOR (exercises 9.1-9.6)

4 December

[E] EXERCISES done by all and discussed as in an ordinary exercise class: 9.7-9.9
9.4 INTERVAL DECOMPOSITION – *read it yourself*
[26] **9.5 THE ENDEMIC STEADY STATE** and
9.6 VACCINATION

10. SPATIAL SPREAD – *we omit this chapter; it overlaps with the course Spatial models of ecology and evolution.*
11. MACROPARASITES – *we omit this chapter.*
12. WHAT IS CONTACT? – *we take selected parts of this chapter as follows*

12.6 NETWORK MODELS
12.6.1 – 12.6.2 *Read these introductory sections yourself*
[27] 12.6.3 A network with hardly any structure

7 December

[28] 12.6.4 An STD in a network with hardly any structure

12.5 STOCHASTIC FINAL SIZE AND MULTI-LEVEL MIXING

12.5.1 Modelling transmission within and between households – *read it yourself*

12.5.2 Comments and extensions of the household epidemic – *read it yourself*

[29] 12.5.3 The distribution of a within-household outbreak

[30] 12.5.4 The final-size distribution of the household epidemic

11 December

No new material, reserved for any last-minute adjustments and independent study.