MATHEMATICAL MODELLING
EXERCISE 8 – 9

8.
Consider the site-competition model from the lectures:

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}X &= \lambda(X_{\max} - X)Y - \delta X && \text{(site owners)} \\ \tfrac{\mathrm{d}}{\mathrm{d}t}Y &= -\lambda(X_{\max} - X)Y + \beta X - \mu Y && \text{(searchers)} \end{aligned}$$

(a) Suppose that Y is small relative to X and that λ is large compared to the other parameters. Scale Y and λ accordingly using the small dimensionless scaling parameter $\varepsilon > 0$. Show that you get a slow-fast system, and analyse the slow and the fast dynamics in the limit of $\varepsilon \to 0$. Compare the equation for the slow dynamics with the logistic equation;

(b) Suppose that Y is large relative to X and that δ and β are large compared to the other parameters. Scale Y and δ accordingly using the small dimensionless scaling parameter $\varepsilon > 0$. Show that you get a slow-fast system, and analyse the slow and the fast dynamics in the limit of $\varepsilon \to 0$. Compare the equation for the slow dynamics with the logistic equation;

9.
Consider the predator-prey model from the lectures:

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}X &= \alpha f(X) - \beta X Y_{\mathrm{S}} && \text{(prey)} \\ \tfrac{\mathrm{d}}{\mathrm{d}t}Y_{\mathrm{S}} &= -\beta X Y_{\mathrm{S}} + \tfrac{1+p}{T}\, Y_{\mathrm{H}} - \delta\, Y_{\mathrm{S}} && \text{(searching predators)} \\ \tfrac{\mathrm{d}}{\mathrm{d}t}Y_{\mathrm{H}} &= +\beta X Y_{\mathrm{S}} - \tfrac{1}{T}\, Y_{\mathrm{H}} - \delta\, Y_{\mathrm{H}} && \text{(handling predators)} \end{aligned}$$

(a) By scaling only parameters, transform this system in a fast-slow system such that $Y := Y_{\mathrm{S}} + Y_{\mathrm{H}}$ (total predator density) becomes a slow variable and X and Y_{H} become fast variables;

(b) Analyse the 2-dimensional fast dynamics using the phase-plane method and local stability analysis;

(c) Analyse the 1-dimensional slow dynamics on the slow manifold.