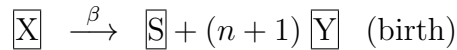


MATHEMATICAL MODELLING
EXERCISE 6 – 7

6.

Consider the following variation on the site-competition model with i-states \boxed{S} (free site), \boxed{X} (occupied site; also: site owner) and \boxed{Y} (individual searching for a free site), and individual-level processes modelled as



- (a) give a more detailed interpretation of the process indicated as “birth”;
- (b) give the corresponding differential equations for the population densities of \boxed{S} , \boxed{X} and \boxed{Y} ;
- (c) show that the total density of sites (i.e., occupied and free) stays constant, and use this to reduce the system to two equations: one for X and one for Y ;
- (d) turn the system into a fast-slow system by assuming that n and μ are large compared to all other parameters. To this end, use a small scaling parameter $\varepsilon > 0$ as we did in the lectures;
- (e) show that in the limit for $\varepsilon \rightarrow 0$, the system becomes essentially one-dimensional as the dynamics becomes confined to a slow manifold. Give the equation for the slow manifold, and show that the slow dynamics on this manifold are given by the logistic equation.

7.

Consider the epidemic model

$$\left\{ \begin{array}{l} \frac{ds}{dt} = -\beta si + \delta r \quad (\text{susceptible but healthy}) \\ \frac{di}{dt} = +\beta si - \gamma i \quad (\text{infected}) \\ \frac{dr}{dt} = +\gamma i - \delta r \quad (\text{recovered and temporally immune}) \end{array} \right.$$

Interpret the different terms on the right hand side in terms of individual level processes. Reduce the dimensionality of the system using a conservation relation. Do a phase-plane analysis and determine the local stability of all equilibria. You may have to consider different cases depending on the parameter values.