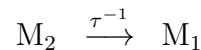
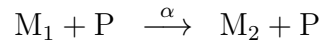


MATHEMATICAL MODELLING
EXERCISE 22 – 23

22.

Suppose the prey can be in one of two states, M_1 (freely roaming about) or M_2 (hiding for the predator), subject to the following transitions only:



Suppose further that prey in state M_1 diffuse, but that prey in state M_2 sit still and thus do not diffuse. The predator, too, diffuses and in addition to that also has a positive taxis towards higher population densities of M_1 , presumably because it does not see the hiding prey.

(a) Give the corresponding system of three (reaction-diffusion) equations for the corresponding population densities m_1 , m_2 and p , subject to reflecting (i.e., zero-flux) boundaries. Simplify the expressions of the boundary conditions as much as possible.

(b) Assuming that the reactions (i.e., state transitions) are very fast compared to diffusion and taxis, calculate the quasi-equilibrium of the fast dynamics. Use this to reduce the number of equations from three to two: one for total prey density $m := m_1 + m_2$ and one for predator density p . Rewrite the boundary conditions accordingly.

(c) Rewrite the equations with a density-dependent diffusion coefficient $D_1(p)$ and density-dependent taxis coefficients $Q_1(p)$ and $Q_2(m, p)$, i.e., rewrite the equations in the form

$$\partial_t m = \partial_x (D_1(p) \partial_x m)$$

$$\partial_t p = D_2 \partial_{xx} p - \partial_x (Q_1(p) p \partial_x m) + \partial_x (Q_2(m, p) p \partial_x p)$$

Give explicit expressions for the density-dependent coefficients.

23.

Consider the predator-prey system

$$\varepsilon \partial_t m = a - b m - \beta m p$$

$$\partial_t p = \gamma \beta m p - \delta p + D \partial_{xx} p$$

Suppose that $0 < \varepsilon \ll 1$ so that the m -dynamics are much faster than the p -dynamics.

(a) Calculate the quasi-equilibrium of the m -dynamics, and use this to reduce the system to a single equation for p only.

(b) Investigate what travelling wave solutions are possible on the infinite domain if $a\gamma\beta > \delta b$.