MATHEMATICAL MODELLING
EXERCISE 19 – 21

19.
Consider the PDE

$$\begin{cases} \partial_t n = an + D\partial_{x,x}n & \text{for } x \in [0, L] \text{ and } t \geq 0 \\ \\ n = 0 & \text{for } x = 0, L \end{cases}$$

for $a, D > 0$.

(a) Give a biological interpretation of the system with the boundary conditions.
(b) Determine the stability of the trivial equilibrium $n = 0$.

20.
Consider the PDE

$$\begin{cases} \partial_t n = rn\left(1 - \frac{n}{K}\right) + D\partial_{x,x}n & \text{for } x \in [0, L] \text{ and } t \geq 0 \\ \\ n[0, t] = 0 \;\&\; \partial_x n[L, t] = 0 \end{cases}$$

for $r, K, D > 0$.

(a) Give a biological interpretation of the system with the boundary conditions.
(b) Determine the stability of the trivial equilibrium $n = 0$.

18.
Consider the PDE

$$\begin{cases} \partial_t n = D\partial_{x,x}n - a\,\partial_x(n\partial_x n) & \text{for } x \in [0, L] \text{ and } t \geq 0 \\ \\ \text{zero flux boundaries} & \text{for } x = 0, L \end{cases}$$

or $a, D > 0$.

(a) Give a biological interpretation of the system with the boundary conditions.
(b) Determine the stability of the positive equilibrium $n = \bar{n}$ (const.) if $\bar{n} < D/a$.
(c) How would the system behave for $\bar{n} < D/a$?