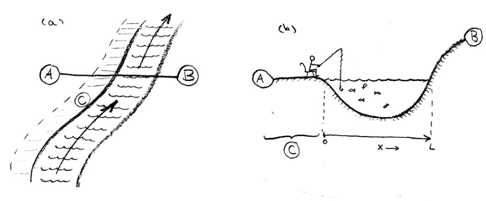
MATHEMATICAL MODELLING
EXERCISE 17 – 18

17.
Consider a situation as in the following figure: the picture on the left shows a map



of a river. The region on the left bank of the river has been labeled C. The picture on the right gives the cross section A–B, and it shows that the river is populated by fish, which are preyed upon by a terrestrial species living in compartment C.

In this exercise we formulate a spatial extension of the predator-prey model of Gause:

$$\begin{cases} \frac{dn}{dt} = g(n) - f(n)p & \text{(prey)} \\ \\ \frac{dp}{dt} = \gamma f(n)p & \text{(predator)} \end{cases}$$

(see page 36 of the lecture notes of 14–2–2012). To this end, let x denote the distance from the left shore as in the picture, and assume that individual fish move randomly. Assume further that compartment C is well-mixed.

(a) Give PDE/ODE equations describing the above situation. **(b)** What boundary conditions would you impose on the fish density? **(c)** What does it mean in terms of (the speed of) individual movement if we assume spatial structure for the fish but not for the predator? **(d)** Re-formulate the model assuming that both the prey and the predator are spatially structured and allowing for both random movement and taxis in both populations.

18.
Consider the following system defined on $[0, L] \times \mathbb{R}_+$:

$$\partial_t n(x,t) = -\partial_x\big(- D\partial_x n(x,t) + \alpha n(x,t)\big)$$

$$0 = -D\partial_x n(x,t) + \alpha n(x,t) = 0 \text{ for } x = 0 \text{ and } x = L$$

(a) Give a biological interpretation of the system including the boundary conditions. **(b)** Give an equilibrium solution of the system. **(c)** Investigate the stability of the equilibrium.