MATHEMATICAL MODELLING
EXERCISE 15 – 16

Introduction – The Nicholson-Bailey host-parasitoid model is given by

$$\begin{cases} x_{n+1} = \alpha x_n e^{-\beta y_n} \\ \\ y_{n+1} = \alpha x_n (1 - e^{-\beta y_n}) \end{cases}$$

where x_n and y_n denote, respectively, the host and parasitoid densities at the beginning of the season in year n. The model can be derived from the within-season dynamics

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}t} v_0(t) = -\beta y_n v_0(t), \quad v_0(0) = \alpha x_n \\ \\ \frac{\mathrm{d}}{\mathrm{d}t} v_1(t) = +\beta y_n v_0(t), \quad v_1(0) = 0 \end{cases}$$

where $v_0(t)$ and $v_1(t)$ denote the population densities of, respectively, un-parasitised and parasitised host larvae, combined with the between-season dynamics $x_{n+1} = v_0(1)$ and $y_{n+1} = v_1(1)$. This is not the only possible way to derive Nicholson-Bailey model, but it is a way that allows for many variations.

15.

As a first variation, assume that host larvae are being produced at a constant *per capita* rate during the season instead of by a single reproductive burst at the beginning of the season.

(a) How does this assumption affect the within-season dynamics as well as the initial conditions? **(b)** Derive the corresponding between-season dynamics $x_{n+1} = v_0(1)$ and $y_{n+1} = v_1(1)$. **(c)** Give conditions for the existence of a positive equilibrium $(\overline{x}, \overline{y})$, and calculate this equilibrium when it exists. **(d)** When is the positive equilibrium stable?

16.

Maybe the most unrealistic characteristic of the Nicholson-Bailey model is that the host population grows without bound when there is no parasitoid. To remedy this, as a second variation of the model, assume that the adult hosts cannibalise on their of larvae as in the Ricker model.

(a) How does this assumption affect the within-season dynamics? **(b)** Derive the corresponding between-season dynamics $x_{n+1} = v_0(1)$ and $y_{n+1} = v_1(1)$ and show that the host and parasitoid densities remain bounded. **(c)** Give conditions for the existence of a positive equilibrium $(\overline{x}, \overline{y})$, and calculate this equilibrium when it exists. **(d)** When is the positive equilibrium stable?