MATHEMATICAL MODELLING
EXERCISE 12 – 14

12.
Suppose that during each season (1) adult mortality is negligible, (2) juveniles are being produced at a constant rate β per adult individual, (3) juveniles die due to juvenile-juvenile interference at a rate γ. At the end of each season all adults die, but some of the juveniles survive to become the adults in the next season.

– Derive a discrete-time population model $x_{n+1} = f(x_n)$ for the population density of adults at the beginning of successive seasons. Calculate the equilibria and determine their stability.

13.
During the lectures we gave two different derivations of the Ricker model

$$x_{n+1} = ax_n e^{-bx_n}.$$

One derivation involved adult-juvenile interference during the season following a single reproductive burst at the beginning of the season. The other derivation involved constant reproduction during the season, followed by random (Poisson) distribution of juveniles over sites with scramble competition within each site.

– How does season length T affects the stability of the Ricker model for each of these two derivations?

14.
Suppose that during each season we have

$$\boxed{\mathrm{U}} \xrightarrow{\mu} \text{dead}$$

$$\boxed{\mathrm{U}} + \boxed{\mathrm{U}} \xrightarrow{\nu} \boxed{\mathrm{U}} + \text{dead}$$

$$\boxed{\mathrm{U}} + \boxed{\mathrm{V}} \xrightarrow{\gamma} \boxed{\mathrm{U}} + \text{dead}$$

$$\boxed{\mathrm{V}} \xrightarrow{\delta} \text{dead}$$

following a single reproductive burst at the beginning of the season. At the end of each season all adults die, but some of the juveniles survive to become the adults in the next season.

– Derive a discrete-time population model $x_{n+1} = f(x_n)$ for the population density of adults at the beginning of successive seasons. Calculate the equilibria and determine their stability.