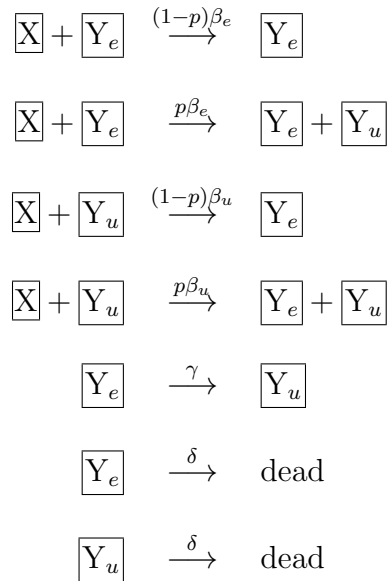


MATHEMATICAL MODELLING
EXERCISE 10 – 11

10.

Consider a predator-prey model with one i-state \boxed{X} for the prey and two i-states $\boxed{Y_e}$ and $\boxed{Y_u}$ for the predator corresponding to individuals that are, respectively, experienced and unexperienced in the subtle art of capturing prey. The i-state transitions of the predator are given by



where $0 < \beta_u < \beta_e$. Assume further that the predator-free dynamics of the prey are given by

$$\frac{d}{dt}X = g(X).$$

- (a) Give an interpretation of each of the above i-level processes in terms of individual behaviour. Give the corresponding p-level equations for the population densities X , Y_e and Y_u ;
- (b) Suppose we want X and $Y := Y_e + Y_u$ (total predator density) to change slowly, while Y_e and Y_u equilibrate fast for given X and $Y := Y_e + Y_u$. To get this done, how should the parameters and/or population densities be scaled?;
- (c) Study the fast dynamics of Y_e and Y_u . At quasi-equilibrium values, what fraction of the predator population is "experienced"?
- (d) Study the slow dynamics of X and Y on the slow manifold assuming some concrete but simple expression for $g(x)$ (your choice).
- (e) Give an explicit expression of the functional response in this model. Sketch its graph;

(f) How would you modify the model on the individual level to get a Holling III functional response?

11.

For each of the following situations, give an explicit expression for the functional response of the predator:

(a) When two predator individuals meet they "pair up" (i.e., they interact with one another) before they go their own way again. Only single (i.e., "unpaired") predators catch prey; The pairing up and breaking up are fast processes.

(b) When two prey individuals meet they pair up for a while before then they break up again. These are fast processes. Only single prey can be attacked by the predator.

(d) When a predator individual meets a prey individual they pair up (i.e., one chases the other, or they fight, or the predator stalks the prey). Most of the time the predator's attack is unsuccessful and they break up again. Relative to actual capture and all other processes the pairing up and breaking up are fast processes.

Note: whenever you think there is incomplete information, you have to make up the missing information, but be explicit about it. This is part of the exercise.