

MATHEMATICAL MODELLING
EXERCISE 1 – 3

1.

Model the i-level process of dying as a Poisson process with rate $\delta > 0$, i.e.,

(a) give a differential equation for the probability $P(t)$ that an individual is still alive at time t ;

(b) solve the equation with initial condition $P(0) = 1$ to show that the lifetime of an individual has an exponential distribution with a mean equal to δ^{-1} ;

(c) give the corresponding differential equation for the population density $N(t)$ of individuals still alive at time t .

2.

Model the following i-level processes as Poisson processes with density-dependent or density-independent rates (whichever is appropriate), and give the corresponding (systems of) differential equations for the population densities:

(a) migration from one habitat to another and back again;

(b) the hatching of an egg;

(c) asexual reproduction;

(d) a male meets a female and together they produce an offspring;

(e) a predator kills its prey;

(f) two individuals on meeting one another and form a pair, but after an exponentially length of time they breakup and go their own way;

(g) two competitors meet and one kills the other;

(Hint: first decide which are the i-states; then decide which are the i-state transitions; finally write down the corresponding population equations.)

3.

Consider two i-states \boxed{A} and \boxed{B} with either $\boxed{A} \xrightarrow{\alpha} \boxed{B}$ or $\boxed{A} \xrightarrow{\alpha A} \boxed{B}$ as, respectively, density-independent or density-dependent state transitions Here A denotes the population density of individuals in state \boxed{A} .

(a) What kind of event could these transitions represent?

(b) For each separately, solve the corresponding differential equations for the population density of \boxed{A} .

(c) For each separately, calculate the expected time till a transition;

(Hint: in (c) first calculate the fraction $A(t)/A(0)$ of the original population still in state \boxed{A} , and then realise that this is also the probability that an individual is still in that state.)