University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 12, 7.12.2015

N.B. The files mentioned in the exercises (if any) are available on the course homepage.

1. Show experimentally that for real 2×2 matrices `A=[a b; c d]` the following equality holds;

$$cond(A) = s + \sqrt{s^2 - 1} \quad \text{where} \quad s = (a^2 + b^2 + c^2 + d^2)/(2|\det(A)|).$$

2. Use the data d122dat.dat to fit the model $f(\lambda_1, \lambda_2, \lambda_3, x) = \lambda_1/(1 + (x - \lambda_2)^2) + 1/(1 + (x - \lambda_3)^2)$. Use the initial values `[1,-1,2]` as a guess. Hint: `parfit` or `parf04`.

3. The program hlp123.m on the www-page plots a curve through given points. Use it to plot the shape of your hand using sufficiently many points, e.g.25-30 points. Experiment with the program by changing pchip to spline and other methods of interpolation.

4. Write a program `numdf`, which computes the numerical derivative of a function at the points in a given vector, using the function `numder`. The program call should be of the form

```
numdf('myf(x)',z, 1e-4)
```

where `z = 0:0.05:1;`, and `myf` is a function. Plot the error of the numerical derivation using the command `pic('cos(x)- numdf(''sin(x)'', x, 1e-4)')`. Hint: The file hlp116.m contains numder and pic.

5. Consider the data $(x_j, y_j), j = 1, ..., m$, and set

$$f(a, b, c, d, x) = ax^2 + bx + c + d/x\,, \quad S = \sum_{j=1}^{m} (y_j - f(a, b, c, d, x_j))^2\,.$$

A researcher is modelling the political awareness in EU countries using this model.

(a) Help the researcher to set up the normal equations. (Recall that these are $\frac{\partial S}{\partial a} = 0$, $\frac{\partial S}{\partial b} = 0$, $\frac{\partial S}{\partial c} = 0$, $\frac{\partial S}{\partial d} = 0$.) Do not solve the normal equations.

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(b) Use the method of problem d105 to write the problem in matrix form $X\lambda = Y$, where $X(j,:) = [x_j^2, x_j, 1, 1/x_j], Y(j,1) = y_j$ and $\lambda = [a;b;c;d]$. Then generate synthetic data and use solve this system of equations $\lambda = X \setminus Y$.

Hints for exam

Typical questions are:

a. Newton's method for solving a system of two equations and two unknowns, one or two iteration steps.

b. Dirichlet's problem

c. LSQ-approximation in the L^2-sense (e.g. approximate $\tan x$ with a second degree polynomial in $[0, \pi/4]$.)

d. Some homework problem from problem sets 7-12.

e. Some easy theory question.

It is not required that you memorize any difficult formulas (such as Fourier-coefficients), those will be given if needed.