University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 11, 30.11.2015

N.B. The files mentioned in the exercises (if any) are available on the course homepage.

1. The Gram–Schmidt method also applies to orthogonalize a system of functions. Use this method to orthogonalize the system $\{1, x, x^2, x^4\}$. of the space C([−1, 1]) with the inner product $(f, g) = \int\limits_{-1}^{1} f(x)g(x)\,dx$.

2. Part (b) of this problem deals with the so called Gibbs phenomenon.

(a) Show that for each fixed x the number $S_n(x) = \frac{nx}{1+n^2x^2}$ approaches zero when n grows to ∞, and find the extremum values of $S_n(x)$ with respect to x. Graph the function $S_n(x)$ when n=2:2:10.

(b) Show that the Fourier series of $f(x) = (\pi - x)/2$ on $(0, 2\pi)$ is $\lim\limits_{n\to\infty} S_n(x)$, where $S_n(x) = \sum_{k=1}^{n} \frac{1}{k} \sin(kx)$. Graph S_n for n=10:2:20 and find graphically the global maximum $x_0 \in (0, 2\pi)$ of $S_n(x)$ and estimate graphically the number $|S_n(x_0) - f(x_0)|/|f(x_0)|$.

The package `gibbs.zip` on the www-page demonstrates Gibb's phenomenon. Recall the Fourier series of a continuous function $f : [0, 2\pi] \to \mathbf{R}$

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx)),$$

where

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)dx,\ n = 0, 1, 2, \ldots$$

and

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)dx,\ n = 1, 2, 3, \ldots$$

3. Find the nth partial sum of the Fourier series of $f(x) = x^2$ on $(0 < x < 2\pi)$ and graph it for n=4:2:10.

4. Consider again the problem of fitting a "line with a break point"to a data set, as in d101. Now, instead of choosing the break point (s, t) with a mouse click as we did in d101, use the method of the program parfit to

FILE: ~/mme/demo15/d11/d11.tex — 2. marraskuuta 2015 (klo 8.34).

find the best break point $(s,t) = (\lambda_1, \lambda_2)$. The object function will be, with the notation of the solution to d101, s1+s2. Apply this optimized version of d101 to the data of d101. Recall that the object function value obtained in d101, after the fitting was 2.62. Do you get a better value this time?

5. An astronomer has the following observations about a comet approaching the Earth.

Taulukko 1: Comet coordinates

x	1.02	0.95	0.87	0.77	0.67	0.56	0.44	0.30	0.16	0.01
y	0.39	0.32	0.27	0.22	0.18	0.15	0.13	0.12	0.13	0.15

Determine the equation of the comet on the basis of this data using a quadratic function

$$ay^2 + bxy + cx + dy + e = x^2.$$

Hint: The problem yields the overdetermined system

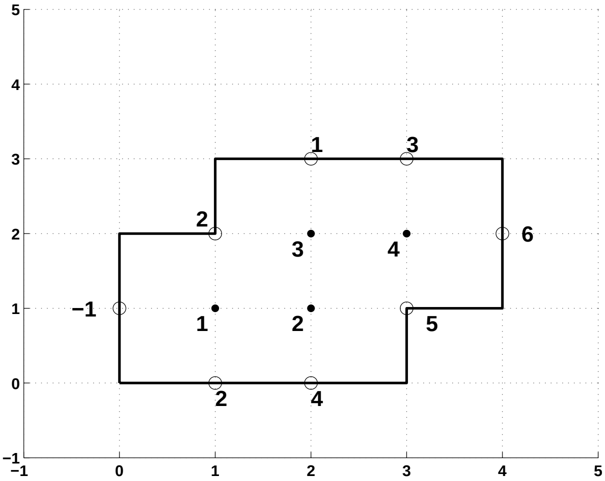
$$ay_i^2 + bx_iy_i + cx_i + dy_i + e = x_i^2\,, i = 1, ..., 10,$$

which we will solve with the LSQ method for the coefficient vector $sol = (a,b,c,d,e)^T$. We rewrite this as $M * sol = w$ and its solution is obtained with $sol = M\backslash w$ (or, alternatively, $sol = pinv(M) * w$).

6. (a) Solve Dirichlet's problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

in the situation pictured below, by using the boundary values and the numbering of variables as in the picture. The sidelength of a square is 1.



(b) As in (a), but in the situation of the following picture.