University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 05, 12.10.2015

N.B. The files mentioned in the exercises (if any) are available on the course homepage.

1. (a) Plot the functions $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2)dy$ and $f(x) = \frac{1}{\sqrt{\pi}} \exp(-x^2)$ on the interval $[0, 3]$. Note that erf(x) is a built-in function of MATLAB. Find the root of the equation $f(x) = erf(x)$ on this interval.

(b) Show by change of variable that for $a, b \in R, a \neq 0, x_1 < x_2,$

$$\int_{x_1}^{x_2} \exp\left(-\frac{(x-b)^2}{(2a^2)}\right) dx = a\sqrt{\frac{\pi}{2}} \left(erf(\frac{x_2 - b}{a\sqrt{2}}) - erf(\frac{x_1 - b}{a\sqrt{2}}) \right) .$$

Verify this also by MATLAB experiments.

2. On the www-page is given the program `hpl052.m` which compares two methods of numerical integration, namely Riemann's sum and Simpson's Rule, over a rectangular region in the plane with the test function $f(x, y) = xy$. The program prints the error.

(a) Modify the program to use the function $g(x, y) = \sin(2x) * \cos(4 * y)$ and report the results.

(b) Write the code also for the Trapezoid Rule and the MATLAB built-in function `dblquad` and report the error. Provide an order or preference of the methods based on the accuracy of each method.

3. Use MATLAB to generate a picture of the Julia set of the iteration $z \mapsto z^2 + a, a = 0.3 - i * 0.2$.

4. Suppose that A is a non-singular $n \times n$ matrix with columns $A^{(j)}, j = 1, ..., n$, and x and b are $n \times 1$ vectors. By Cramer's Rule, the solution to $Ax = b$ is given by

$$x_j = ((\det(A))^{-1}) \det(C_j) \, , C_j = [A^{(1)}A^{(2)}...A^{(j-1)}bA^{(j+1)}...A^{(n)}] \, .$$

Verify this procedure with MATLAB tests for small n . For how big values of n this is a reasonable procedure?

5. The daily measurement data of a the body temperature of a patient are stored in files a1.txt,..., a7.txt in the format one measurement/line.

FILE: ~/mme/demo15/d05.tex — 29. elokuuta 2015 (klo 11.08).

Write a program that reads the measurements and plots a histogram (the command bar and hist may be useful here) of the results and computes the mean temperature.

6. Theorem 1.2.2 on p. 12 of P. Borwein-T. Erdélyi: Polynomials and Polynomial Inequalities Springer-Verlag, 1995 states that if $p(z) = a_n z^n + a_{n-1}z^{n-1} + ... + a_0$ and $a_0 \geq a_1 \geq ... \geq a_n > 0$ then all zeros of p lies outside the open unit disk. Verify experimentally this statement by generating random coefficients a_j and by plotting the roots in the plane.