University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 03, 28.9.2015

N.B. The files mentioned in the exercises (if any) are available on the course homepage

1. Suppose that the coefficients of a polynomial are known (for instance, generate 10 polynomials with random coefficients). Find the roots with the command "roots"and write the coefficients and the the real and imaginary parts of the roots in a file. Also plot the graph of the function.

2. (a) Prove that $\int_a^b \int_c^d xy\,dx\,dy = (d^2 - c^2)(b^2 - a^2)/4$ for $b > a, d > c$.

(b) Use the MATLAB function doubleint0.m on the www-page to compute this integral when $(a, b, c, d) = (0, 3, 0, 2)$ and tabulate the difference exact value minus numerical value when the number $m[n]$ of subdivisions in the $x[y]$ direction has the value $m = 10 : 20 : 90, n = 10 : 20 : 90$. You may do this as follows

```
exact=  (d^2- c^2)*(b^2 -a^2)/4;
for m=10:20:90
for n=10:20:90
    numer=doubleint0(a,b,c,d,m,n);
    fprintf('  %12.3e', numer-exact);
end
fprintf('\n')
end
```

3. The eigenvalues (=characteristic roots) of an $n \times n$ complex matrix $(a_{i,j})$ lie in the closed region of the z-plane consisting of all the disks

$$\{z \in \mathbb{C} : |a_{i,i} - z| \leq \sum_{j=1, j \neq i}^{n} |a_{i,j}|\}, j = 1, ..., n .$$

These are so called Gerschgorin disks. Check the validity of this statement as follows:

(a) For each n=3:3:18 generate a random complex $n \times n$ matrix and compute its eigenvalues.

FILE: ~/mme/demo15/d03/d03.tex — 29. elokuuta 2015 (klo 14.37).

(b) For each case plot the Gerschgorin disks and check visually that the statement holds.

4. The fixed point method for numerical solution of $f(x) = x$ when $f : \mathbb{R} \to \mathbb{R}$ is based on the fixed point iteration $x_{n+1} = f(x_n)$. This converges for all $x_0 \in [a, b]$ if there exists $c \in (0, 1)$ such that $|f'(x)| < c$. We want to use this method to find the root of $1 - x - \sin x = 0$. Therefore we introduce an auxiliary function

$$g(x) = x + (1 - x - \sin x)/\lambda$$

and define the sequence $x_{n+1} = g(x_n)$. Show that if $\lim x_n = w$ then w satisfies the equation $1 - w - \sin w = 0$. (a) Plot the function $1 - x - \sin x$ and find a guess w for the root. Also choose a guess for the interval $[a, b]$ using w and make sure that a point s with $g(s) = s$ exists in $[a, b]$. (b) Differentiate g and find $\lambda \in (0, 1)$ such that $|g'(x)| < c$ for some $c \in (0, 1)$ and all $x \in [a, b]$. In this step we may need to replace $[a, b]$ by a smaller interval containing w (and make sure that a point s with $g(s) = s$ exists in this smaller interval). (c) Finally use the method with this fixed λ and $[a, b]$ and choose $x_0 \in [a, b]$ to start the iteration for g and find the root.

5. A $m \times n$ matrix $A = (a_{ij})$ is called upper triangular if $a_{ij} = 0$ whenever $i > j$. Generate upper triangular 7×7 matrices and study experimentally whether (a) the product of two such matrices is again upper triangular, (b) an upper triangular matrix has an upper triangular matrix as the inverse (c) whether the determinant is always nonzero. Write a program to solve the upper triangular linear $n \times n$ system of equations.

6. Suppose that the random points a, b, c, d are located on the unit circle in such a way that b and c are on the smaller arc between a and d. Show by MATLAB tests that the point of intersection of the segments $[a, b]$ and $[c, d]$ (or of the lines containing these segments) is $(ab(c+d) - cd(a+b))/(ab - cd)$. Hint: The points a, b, c, d should be considered as complex numbers.