University of Helsinki / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 02, 21.9.2015

1. A data set $(x_j, y_j), j = 1, ..., m,$ and a fixed point (s, t) is given. Consider the lines $y = t + k(x - s)$ through this point and for varying parameter k form the sum

$$A(k) = \sum_{j=1}^{m} (y_j - t - k(x_j - s))^2 \,.$$

Write the formula for the derivative $A'(k)$, and solve the linear equation $A'(k) = 0$ for k. With this particular value of k we get the least square (LSQ) line fitted to our data set through the point (s, t). Generate some synthetic data [e.g. $x_j = 0.5 * j, y_j = 0.3 * x_j + 0.1 * \sin(30 * x_j), j = 1, ..., 20,$] plot the data and the LSQ-line through the point $(0, 0)$ and check visually whether the line fits nicely to the data set.

2. P. J. Myrberg (1892–1976) has published the following algorithm for the computation of the square root in his paper in Ann. Acad. Sci. Fenn. Ser. A I 253 (1958), 1-19. Fix $z \in \mathbb{C} \setminus \{1\}$ and let $q_0 = (z+1)/(z-1), q_{k+1} = 2q_k^2 - 1, k = 0, 1, ...$ Then $\sqrt{z} = \Pi_{k=0}^{\infty}(1 + 1/q_k)$. Carry out MATLAB tests to check this claim in the following cases

(a) z is real and > 1,

(b) z is real and in $[0, 1]$.

(c) z is complex [Hint: Generate a random complex number w and set $z = w * w$ and see whether the algorithm gives you w.]

3. The use of the Trapezoid formula for numerical integration of a tabulated function $f(x_i) = y_i$ is based on the formula

$$s = \sum_{i=1}^{n} (x_{i+1} - x_i)(y_{i+1} + y_i)/2,$$

where the tabulated values are $(x_i, y_i), i = 1, \ldots, n + 1$, and $x_i < x_{i+1}$.

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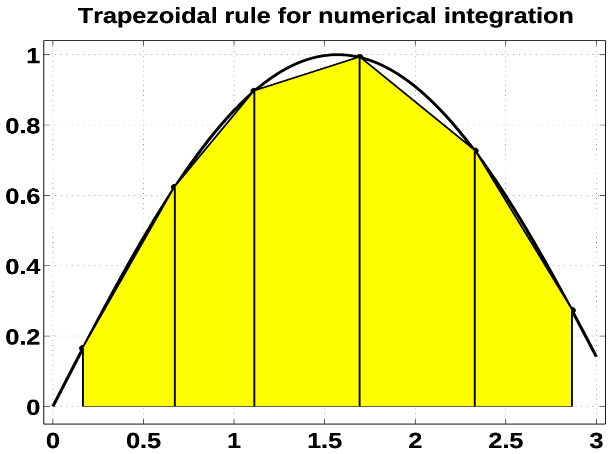

(a) Show that if $f(x) = \sum_{j=1}^{m} c_j \sin(d_j * x)$, then by high school calculus

$$\int_a^b f(x)dx = \sum_{j=1}^{m} (c_j/d_j)(\cos(d_j * a) - \cos(d_j * b)).$$

(b) Generate coefficient vectors `c` and `d` [e.g. `c=3*rand(1,5);` `d= 1+10*rand(1,5)`] and use the exact formula in part(a) to compute the integral $\int_0^1 f(x)dx$ and also compute the value by the trapezoid formula and print the error when the points $x_j = (1/n) * j, j = 0, 1, ..., n$, are used in the trapezoid formula with $n = 20 : 10 : 200$.

4. For random points a, b, c, d in the plane, find the point of intersection of the lines L_1 through a, b and L_2 through c, d. Plot the picture of the lines and the point of intersection.

5. Fix $a > 0$. It is well-known that if $0 < x_0 < 2/a$ the recursive sequence defined by $x_{k+1} = x_k(2 - ax_k)$ converges to $1/a$.

(a) Write a MATLAB function to compute the reciprocal with this method.

(b) Apply the method to several test cases (say to $0.1 * k, k = 1 : 1000$,) and report the error.

6. Numerical analysis textbooks often point out that the quadratic formula $x_{1,2} = (-b \pm \sqrt{b^2 - 4ac})/(2a)$ for the solution of the equation $ax^2 + bx + c = 0$ may lead to distorted results when $|4ac|$ is very small. Plan a MATLAB experiment to justify this remark and report the errors in the test cases you have used. (Hint. Choose $a = b^2/(4c) \cdot 10^{-m}$, $m = 3, \ldots, 15$. Compare your result either to what you get from the method explained during the lectures or from MATLAB `roots([a b c])`.)