

University of Helsinki / Department of Mathematics and Statistics
 SCIENTIFIC COMPUTING
 Exercise 01, 14.9.2015

N.B. The files mentioned in the exercises (if any) are available on the course homepage

1. Apply the recursion formula $x_0 = 1, x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}), n = 0, 1, 2, \dots$ for \sqrt{a} to compute $\sqrt{3}$. Print the results in the following format:

```
n  x(n)  Error
0  1
.....
6  ...
```

2. Approximations to the number π are given by the formula

$$p(n) = \sum_{k=0}^n \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right).$$

Print the first few results in the same format as in problem 1.

3. According to an Internet page, the center w of a circle through three points a, b, c in the complex plane can be found as follows in MATLAB notation:

```
u=(b-c).*abs(a).^2 + (c-a).*abs(b).^2 + (a-b).*abs(c).^2;
v=(b-c).*conj(a)+ (c-a).*conj(b)+ (a-b).*conj(c);
w= u./v;
```

Write a MATLAB script to check this claim. (Hint: Take three random points on the unit circle, then compute w and show that it is 0.)

4. Let $(x_j, y_j), j = 0, 1, \dots, n$ be the vertices of a polygon with $(x_0, y_0) = (x_n, y_n)$. The area of the polygon is given by $a = \frac{1}{2} \sum_{i=1}^n t_i$ with $t_i = x_{i-1}y_i - x_iy_{i-1}$. Carry out the following steps for each of the regular polygons triangle, square and hexagon:

- (a) Choose vertices and compute the area by school geometry.
- (b) Compute the area by the formula and compare to the exact value.
- (c) Plot the figure.

5. Hilbert's inequality says that for $a_k, b_k \geq 0$

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{a_m b_n}{m+n+1} \leq \pi \left(\sum_{m=0}^{\infty} a_m^2 \right)^{1/2} \left(\sum_{n=0}^{\infty} b_n^2 \right)^{1/2} .$$

Carry out a numerical verification of this inequality.

6. What does the following program do? Execute it and interpret the results.

```
% FILE d016.m begins.
for pp=1:3
a=2*rand; b=3*(a+1);
f=@(x)(a*sin(b*x)); v=quad(f,0,1);
exact=(a/b)*(1-cos(b));
fprintf(' %6.4f %6.4f %12.6f %12.4e\n', a, b, v, v-exact)
end
% FILE d016.m ends.
```