Logic Toolbox
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Exercise 1 (warm-up exercises)

1. Let $(A, <)$ be a linearly ordered set. Show that A is well-ordered if and only if there is no infinite descending sequence in A.

2. The Axiom of Foundation states that every nonempty set A has an element x such that $x \cap A = \emptyset$. Show that the Axiom of Foundation implies that no set is a member of itself.

3. Show that if α and β are ordinals then

$$\alpha \subseteq \beta \text{ if and only if } (\alpha \in \beta \text{ or } \alpha = \beta).$$

4. Prove that $\omega = \sup\{0, 1, 2, \dots\} = \bigcup\{0, 1, 2, \dots\} = \{0, 1, 2, \dots\}$.

More generally, if α is a limit ordinal, show that $\alpha = \sup\{\beta : \beta < \alpha\} = \bigcup\{\beta : \beta < \alpha\} = \{\beta : \beta < \alpha\}$. Which equalities hold also for successors?

5. Construct a well-ordering on ω that is different from the usual ordering $\in$ on ω.

6. Show that if R and R^{-1} are both well orderings of the same set X, then X is finite.