1. Kontraktioperiaate $\Rightarrow$ $\{A X_n\}$:llä SPP, VF $= \underline{I}$,

$$\underline{I}(x) = (A\Lambda^*)(x) = \inf\{\Lambda^*(y) \mid Ay = x\}, \quad x \in \mathbb{R}^{d'};$$

Harj. 4, teht. 2 $\Rightarrow$ I on konveksi, samoin siis $\underline{I}$. (lemma 3.8).

2. Varadhanin lemma $\Rightarrow$

$$\Lambda(t) = \sup_{x \in \mathbb{R}^d} \{\langle t, x\rangle - I(x)\} = I^*(t)$$

Lemma 3.10 $\Rightarrow$ $I = (I^*)^* = \Lambda^*$.

3. Jos x_0 ja f ovat kiinteitä, niin Varadhanin lemman nojalla

$$\Lambda_f = \sup_{x \in \mathbb{R}^d} \{f(x) - I(x)\} \geq f(x_0) - I(x_0).$$

Siis

$$I(x_0) \geq f(x_0) - \Lambda_f, \quad \forall f \in C_b(\mathbb{R}^d)$$

$$\Rightarrow I(x_0) \geq \sup\{f(x_0) - \Lambda_f \mid f \in C_b(\mathbb{R}^d)\}.$$

4. GE $\Rightarrow$ $\{X_n\}$:llä SPP, VF $= \Lambda^*$. Varadhanin lemma $\Rightarrow$

$$\lim n^{-1} \log \mathbb{E}(e^{-X_n^2/n}) = \sup_{x \in \mathbb{R}} \{-x^2 - \Lambda^*(x)\} = -\tfrac{1}{5},$$

sillä $\Lambda^*(x) = \frac{1}{4}(x+1)^2$.

5. $$\underline{\lim}\, n^{-1} \log \mathbb{P}(f(X_n) \in G') = \underline{\lim}\, n^{-1} \log \mathbb{P}(X_n \in f^{-1}(G'))$$

$$\geq -\inf\{I(x) \mid x \in f^{-1}(G)\} = -\inf\{J(y) \mid y \in G\},$$

missä J on kuten lauseessa 5.1.
Sama pätee, kun J korvataan $\underline{J}$:llä, $I' = \underline{J}$.
(lause 2.4 ja sen todistus).