Suurten poikkeamien harj. 6, 25.11.-15

1. Jos $\xi \sim \text{Poisson-}a$, niin

$$\mathbb{E}(e^{t\xi}) = \sum_{k=0}^{\infty} e^{-a} \frac{a^k}{k!} e^{tk} = e^{a(e^t-1)}$$

$$\Rightarrow \begin{cases} \Lambda_n(t) = n^{-1} \log e^{\lambda_n n (e^t-1)} = \lambda_n (e^t - 1) \\ \Lambda(t) = \lambda(e^t-1) \end{cases}$$

GE $\Rightarrow$ $\{n^{-1} Y_n\}$:llä SPP, vf $= \Lambda^*$,

$$\Lambda^*(x) = \begin{cases} x \log x - x \log \lambda - x + \lambda, & x \geq 0 \\ \infty, & x < 0 \end{cases}$$

2. $$\mathbb{E}(e^{\langle t, Y_n \rangle}) = \mathbb{E}\left(e^{t_1 \sum_{k=1}^n 1(\xi_k = a_1) + \dots + t_d \sum_{k=1}^n 1(\xi_k = a_d)}\right)$$

$$= \prod_{k=1}^n \mathbb{E}\left(e^{t_1 1(\xi_k = a_1) + \dots + t_d 1(\xi_k = a_d)}\right)$$

$$= \left(\sum_{k=1}^d \mathbb{P}(\xi = a_k) e^{t_k}\right)^n$$

$$\Rightarrow \Lambda(t) = \log \sum_{k=1}^d \mathbb{P}(\xi = a_k) e^{t_k} = \log \sum_{k=1}^d p_k e^{t_k}$$

GE: $\{Z_n\}$:llä SPP, vf $= \Lambda^*$.

3. $$\nabla \Lambda(t) = \left(\frac{p_1 e^{t_1}}{\sum p_k e^{t_k}}, \dots, \frac{p_d e^{t_d}}{\sum p_k e^{t_k}}\right) = (x_1, \dots, x_d)$$

$$\Rightarrow x_i = \frac{p_i e^{t_i}}{\sum p_k e^{t_k}}.$$

Voidaan ottaa $t_i = \log \frac{x_i}{p_i}$ (koska $x_1 + \dots + x_d = 1$).

Siis

$$\Lambda^*(x) = \sum_{i=1}^d x_i \log \frac{p_i}{x_i} - \log\left(\sum_{i=1}^d p_i \frac{x_i}{p_i}\right)$$

$$= \sum_{i=1}^d x_i \log \frac{p_i}{x_i}$$

4. Koska $\nabla\Lambda(0)$ $\exists$, niin $\Lambda^*(x) = 0 \iff$

$$x = \nabla\Lambda(0)$$

(seuraus 3.2). Siis ainut tyypillinen piste on

$x = (p_1, \ldots, p_d)$ (Sin jakauma).

5. Selvästi

$$b_n = \alpha m_{n-1} + (1-\alpha)\alpha m_{n-2} + \ldots + (1-\alpha)^{n-2}\alpha m_1 + (1-\alpha)^{n-1} b_1$$

$$\Rightarrow \begin{cases} \mathbb{E}(e^{tY_n}) = \mathbb{E}\left(e^{t\sum_{k=1}^{n} b_k^{(n)} m_k}\right) e^{-\alpha^{-1}(1-(1-\alpha)^n) b_1 t - n g t}, \\ b_n = 1 - \alpha\left(1 + (1-\alpha) + \ldots + (1-\alpha)^{n-k-1}\right) \end{cases}$$

$$\Rightarrow n^{-1} \log \mathbb{E}(e^{tY_n})$$

$$= \sum_{k=1}^{n} c\left(t\left(1 - \frac{\alpha(1-(1-\alpha)^{n-k})}{\alpha}\right)\right) n^{-1}$$

$$- n^{-1}\alpha^{-1}(1-(1-\alpha)^n) b_1 t - g t$$

$$= \sum_{k=1}^{n} c\left(t(1-\alpha)^{k-1}\right) n^{-1} - g t + o(1)$$

$\longrightarrow -g t$, koska $c(t(1-\alpha)^{n-k}) \to 0$

$\Rightarrow R = \infty$